

Algebra 10.1 Infinite Sequences & Summation Notation

Find the first three terms of the following sequences & the 8th term

① $\{5 - 2n\}$ for $n = 1, 2, 3, \dots$

$$a_1 = 5 - 2(1) = 3$$

$$a_2 = 5 - 2(2) = 1$$

$$a_3 = 5 - 2(3) = -1$$

$$a_8 = 5 - 2(8) = -11$$

② $\{1 + (-1)^{n+1}\}$

$$a_1 = 1 + (-1)^{1+1} = 1 + 1 = 2$$

$$a_2 = 1 + (-1)^{2+1} = 1 + (-1)^3 = 1 - 1 = 0$$

$$a_3 = 1 + (-1)^{3+1} = 1 + (-1)^4 = 1 + 1 = 2$$

$$a_8 = 1 + (-1)^{8+1} = 1 + (-1)^9 = 1 - 1 = 0$$

③ $\left\{\frac{2^n}{n^2+2}\right\}$

$$a_1 = \frac{2^1}{1^2+2} = \frac{2}{3}$$

$$a_2 = \frac{2^2}{2^2+2} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$$

$$a_3 = \frac{2^3}{3^2+2} = \frac{8}{9+2} = \frac{8}{11}$$

$$a_8 = \frac{2^8}{8^2+2} = \frac{256}{64+2} = \frac{256}{66} = \frac{128}{33}$$

④ $\{(-1)^n(2n+1)\}$

$$a_1 = (-1)^1(2(1)+1) = -3$$

$$a_2 = (-1)^2(2(2)+1) = 5$$

$$a_3 = (-1)^3(2(3)+1) = -7$$

$$a_8 = (-1)^8(2(8)+1) = 17$$

⑤ $\{7\}$

$$a_1 = 7, a_2 = 7, a_3 = 7, a_8 = 7$$

Algebra 10, 1

Match

a_n is the number of decimal places in $(0.1)^n$ $\rightarrow (0.1)^3 = .001$
 a_n is the number of positive integers less than n^2 $\times a_3 = 8$
 $\rightarrow 3^2 = 9$ $a_3 = 3$

Recursively defined sequences - you must use previous terms to find the next term!

Find the third term in each of the following

① $a_1 = 2$ $a_{k+1} = 3a_k + 5$

$$a_2 = 3(2) + 5 = 11$$

$$a_3 = 3(11) + 5 = \boxed{38}$$

② $a_1 = 5$ $a_{k+1} = (k+1)a_k$

$$a_2 = 2(5) = 10$$

$$a_3 = 3(10) = \boxed{30}$$

③ $a_1 = 2$ $a_{k+1} = (a_k)^{k+2}$

$$a_2 = (2)^3 = 8$$

$$a_3 = (8)^4 = 8^4 = 4096$$

$$k+1=2$$

$$k+2=3$$

④ $a_1 = 2$ $a_2 = 3$ $a_{k+1} = -2a_k + a_{k-1}$

$$a_3 = -2a_2 + a_1 = -2(3) + 2 = -6 + 2 = -4$$

$$a_4 = -2a_3 + a_2 = -2(-4) + 3 = 8 + 3 = 11$$

Fibonacci Sequence 1, 2, 3, 5, 13, 21, ...

$$a_1 = 1 \quad a_2 = 2 \quad a_{k+1} = a_k + a_{k-1}$$

$$a_3 = 2 + 1 = 3, \quad a_4 = 3 + 2 = 5, \quad a_5 = 5 + 3 = 8, \quad a_6 = 8 + 5 = 13$$

Algebra 10.1

Sequence of Partial Sums

Find the first three terms of the sequence of partial sums for the following.

① $\{5-2n\}$

$$a_1 = 5 - 2(1) = 3 \quad s_1 = 3$$

$$a_2 = 5 - 2(2) = 1 \quad s_2 = 3 + 1 = 4$$

$$a_3 = 5 - 2(3) = -1 \quad s_3 = 3 + 1 - 1 = 3$$

② $\{3 + \frac{1}{2}n\}$

$$a_1 = 3 + \frac{1}{2}(1) = \frac{7}{2} \quad s_1 = \frac{7}{2}$$

$$a_2 = 3 + \frac{1}{2}(2) = 4 \quad s_2 = \frac{7}{2} + 4 = \frac{15}{2}$$

$$a_3 = 3 + \frac{1}{2}(3) = \frac{9}{2} \quad s_3 = \frac{7}{2} + 4 + \frac{9}{2} = \frac{24}{2} = 12$$

③ $\{n\}$

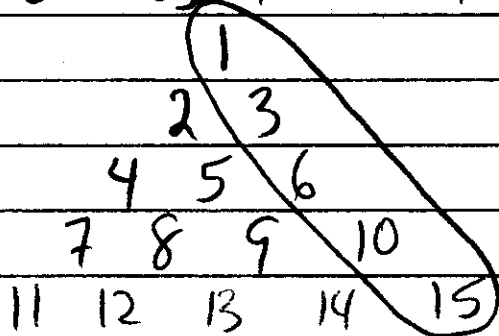
$$a_1 = 1 \quad s_1 = 1$$

$$a_2 = 2 \quad s_2 = 1 + 2 = 3$$

$$a_3 = 3 \quad s_3 = 1 + 2 + 3 = 6$$

$$a_4 = 4 \quad s_4 = 1 + 2 + 3 + 4 = 10$$

$$a_5 = 5 \quad s_5 = 1 + 2 + 3 + 4 + 5 = 15$$



Algebra 10.1

Summations

$\sum_{k=1}^5$ means summation from $k=1$ to 5

$$\textcircled{1} \sum_{k=1}^5 (2k-7) = -5-3-1+1+3 = -5$$

$$\textcircled{2} \sum_{k=1}^4 (k^2-5) = -4-1+4+11 = 10$$

$$\textcircled{3} \sum_{k=1}^6 [1+(-1)^k] = 0+2+0+2+0+2 = 6$$

$$\textcircled{4} \sum_{k=0}^4 k(k-2) = 0-1+0+3+8 = 10$$

$$\textcircled{5} \sum_{k=1}^5 3 = 3+3+3+3+3 = 15$$

$$\textcircled{6} \sum_{k=0}^5 3 = 3+3+3+3+3+3 = 18$$

$$\textcircled{7} \sum_{k=3}^7 -4 = -4-4-4-4-4 = -20$$

$$\textcircled{8} \sum_{k=118}^{147} \frac{1}{3} = \frac{1}{3}(29+1) = \frac{1}{3}(30) = 10$$