Algebra 10.2 Arithmatic Sequences

Ex. $-3, 4, 11, 18, \ldots$ $d = 7$ $7n - 10$ — definition

$+7 \; +7 \; +7$

$a_1 = 7(1) - 10 = -3$

EX $53, 47, 41, 35, \ldots$ $d = 6$ $-6n + 59$

$a_1 = -6(1) + 59 = -6 + 59 = 53$

General Formula for finding the n^{th} term of an arithmatic sequence

$a_n = a_1 + (n-1)d$

EX $10, -2, -14, -26, \ldots$

$a_1 = 10$ and $d = -12$

$a_n = 10 + (n-1) - 12 = 10 - 12n + 12 = 22 - 12n$

EX Find the 5^{th}, 10^{th}, and n^{th} term of the arithmatic sequence

$7, 11, 15, 19, \ldots$ $a_1 = 7$ and $d = 4$

$a_n = a_1 + (n-1)d$

$a_n = 7 + (n-1)4$

$a_n = 7 + 4n - 4$

$a_n = 3 + 4n$

$a_5 = 3 + 4(5) = 23$

$a_{10} = 3 + 4(10) = 43$

Ex Find the 8^{th}, 20^{th}, & n^{th} term of the arithmatic sequence

$-4, -0.5, 3, 6.5, \ldots$ $a_1 = -4$ and $d = 3.5$

$a_n = a_1 + (n-1)d = -4 + (n-1)3.5 = -4 + 3.5n - 3.5 = -7.5 + 3.5n$

$a_8 = -7.5 + 3.5(8) = -7.5 + 28 = 20.5$

$a_{20} = -7.5 + 3.5(20) = -7.5 + 70 = 62.5$

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EX Given $a_4 = 15$, $a_{11} = 43$ find a_n

one solution (slope) $d = \frac{43-15}{11-4} = \frac{28}{7} = 4$

if $a_4 = 15$, then $a_3 = 11$, $a_2 = 7$, $a_1 = 3$

another solution $a_4 = a_1 + (4-1)(4)$

$15 = a_1 + 12$

$a_1 = 3$

$a_n = 3 + (n-1)4 = 3 + 4n - 4 = 4n - 1$

EX Given $a_8 = 41$, $a_9 = 46$, find a_n

$d = 5$

$a_8 = a_1 + (8-1)5$

$41 = a_1 + 35$

$a_1 = 6$

$a_n = 6 + (n-1)5$

$a_n = 6 + 5n - 5$

$a_n = 1 + 5n$

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

EX $a_n = 43 - 3n$ Find S_{26}

$S_{26} = \frac{26}{2}(a_1 + a_{26})$

$= 13((43 - 3(1)) + (43 - 3(26)))$

$= 13(40 - 35)$

$= \boxed{65}$

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EX Find the sum of the arithmetic sequence that satisfies the following conditions. $a_7 = -\frac{8}{3}, d = -\frac{2}{3}, n = 15$

$a_n = a_1 + (n-1)d$

$a_7 = a_1 + (7-1)(-\frac{2}{3})$

$-\frac{8}{3} = a_1 - \frac{12}{3}$

$a_1 = \frac{4}{3}$

$a_n = \frac{4}{3} + (n-1)(-\frac{2}{3})$

$a_n = \frac{4}{3} - \frac{2}{3}n + \frac{2}{3}$

$a_n = 2 - \frac{2}{3}n$

$S_{15} = \frac{15}{2}(a_1 + a_{15})$

$S_{15} = \frac{15}{2}(\frac{4}{3} - 8)$

$S_{15} = \frac{15}{2}(-\frac{20}{3})$

$S_{15} = 5(-10)$

$S_{15} = \boxed{-50}$

EX Express the sum in terms of summation notation

$11 + 16 + 21 + 26$

$d = 5 \quad a_1 = 11 \quad a_n = 5n + 6 \qquad \boxed{\sum_{n=1}^{4} (5n+6)}$

EX Express the sum in terms of summation notation

$-4, -9, -14, -19, -24$

$d = -5 \quad a_1 = -4 \quad a_n = -5n + 1 \qquad \boxed{\sum_{n=1}^{5} (-5n+1)}$

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EX Express the sum in terms of summation notation

$1+3+5+\ldots+73$

$d=2 \quad a_1=1 \quad a_n=2n-1$

$2n-1=73$

$2n=74$

$n=37$

$$\boxed{\sum_{n=1}^{37} 2n-1}$$

EX Express the sum in terms of summation notation

$\frac{3}{7}+\frac{6}{11}+\frac{9}{15}+\frac{12}{19}$

numerator : $d=3 \quad a_1=3 \quad a_n=3n$

denominator: $d=4 \quad a_1=7 \quad a_n=4n+3$

$$\boxed{\sum_{n=1}^{4} \frac{3n}{4n+3}}$$

EX $\sum_{n=5}^{19} 2n+3$

\# of terms (19-5+1)

$$\frac{(15)(a_5+a_{19})}{2}=\frac{15(13+41)}{2}=\frac{15(54)}{2}=\frac{810}{2}=\boxed{405}$$

EX $\sum_{n=3}^{12} 5-2n$

$$\frac{(10)(a_3+a_{12})}{2}=\frac{10(-1-19)}{2}=\frac{10(-20)}{2}=\frac{-200}{2}=\boxed{-100}$$

iLrn 10.2 Part 3

Find the number of terms in the arithmatic sequence with the given conditions:
$a_1 = 8, \quad d = \frac{1}{4}, \quad S = -132$

$S_n = \frac{n}{2}(a_1 + a_n)$
$S_n = \frac{n}{2}(-8 + a_1 + (n-1)d)$
$-132 = \frac{n}{2}(-8 - 8 + (n-1)\frac{1}{4})$
$-132 = \frac{n}{2}(-16 + \frac{1}{4}n - \frac{1}{4})$
$-132 = \frac{n}{2}(-\frac{65}{8} + \frac{n}{8})$
$-1056 = -65n + n^2$
$n^2 - 65n + 1056 = 0$
$(n-32)(n-33) = 0$ try 32, 33

iLrn 10.2 Part 5

A contest will have five cash prizes totalling \$10,000, with a \$200 difference between successive prizes. Find the first prize.

$S_n = \frac{n}{2}(a_1 + a_n)$
$S_n = \frac{n}{2}(a_1 + a_1 + (n-1)d)$
$10{,}000 = \frac{5}{2}(2a_1 + (5-1)(200))$
$10{,}000 = \frac{5}{2}(2a_1 + 800)$
⋮
$a_1 = 1600$
a_5 is first place, since these are in successive order
$a_5 = 1600 + (5-1)200$
$= 1600 + 800 = \boxed{\$2400}$