Algebra 10.5 The Binomial Theorem

Ex $(2x-3y)^3 = (2x-3y)(2x-3y)(2x-3y)$

$= (4x^2-12xy+9y^2)(2x-3y)$

$= 8x^3-24x^2y+18xy^2-12x^2y+36xy^2-27y^3$

$= 8x^3-36x^2y+54xy^2-27y^3$

Binomial Thm

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Pascals Triangle

row 0: 1

row 1: 1 1

row 2: 1 2 1

row 3: 1 3 3 1

row 4: 1 4 6 4 1

row 5: 1 5 10 10 5 1

row 6: 1 6 15 20 15 6 1

Ex: You have 6 friends but only 4 can be in your wedding. How many combinations can you have?

row 6 → $\binom{6}{4} = 15$

entry 4 →

Ex Expand $(2x-3y)^3$ using Binomial Thm.

$(2x-3y)^3 = 1(2x)^3(-3y)^0 + 3(2x)^2(-3y)^1 + 3(2x)^1(-3y)^2 + 1(2x)^0(-3y)^3$

from Pascal's triangle

$= 8x^3-36x^2y+54x^2y^2-27y^3$

Alg 10.5

EX $(\frac{1}{2}c + d^3)^4$

1 4 6 4 1

$= \underline{1}(\frac{1}{2}c)^4(d^3)^0 + \underline{4}(\frac{1}{2}c)^3(d^3)^1 + \underline{6}(\frac{1}{2}c)^2(d^3)^2 + \underline{4}(\frac{1}{2}c)^1(d^3)^3 + \underline{1}(\frac{1}{2}c)^0(d^3)^4$

$= \frac{1}{16}c^4 + 4(\frac{1}{8})c^3 + 6(\frac{1}{4})c^2d^6 + 2cd^9 + d^{12}$

$= \frac{1}{16}c^4 + \frac{1}{2}c^3 + \frac{3}{2}c^2d^6 + 2cd^9 + d^{12}$

EX $(2x - y)^5$

1 5 10 10 5 1

$= 1(2x)^5 + 5(2x)^4(-y)^1 + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)^1(-y)^4 + 1(-y)^5$

$= 32x^5 + 5(16x^4)(-y) + 10(8x^3)(y^2) + 10(4x^2)(-y^3) + 10xy^4 - y^5$

$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

EX Find the term that has a^2b in $(3a - 4b)^3$

$3(3a)^2(-4b)^1 = 3(9a^2)(-4b) = \boxed{-108a^2b}$