2.2 Applied Problems

A Simple Guideline for Problem Solving in One Variable

1. Read through the problem and make sure you understand what it is asking.
2. Draw a picture where applicable.
3. Declare a variable (x, or something) for the thing you are trying to find.
4. Set up an equation using the information you have been given (you can almost guarantee that everything that has been given must be used somewhere).
5. Solve the equation for the variable.
6. Does your answer make sense? Check your answer by plugging it back in.

Ex: A student in Calculus has test scores of 72, 84, 90, and 78. What score on the next test will give the student an average of 82?

let X = 5th test score

$\frac{72+84+90+78+X}{5} = 82$ $\quad (5)\left(\frac{324+X}{5}\right) = (82)5$ $\quad 324+X=410$ $\quad X=86$

Ex: Before the final exam, Sara has test scores of 71, 79. 82, 76, and 84. If the final exam counts as 1/3 of her grade, what does she have to get in order to have a 75 average in the class?

let X = final exam grade

$\frac{71+79+82+76+84}{5} = 78.4$ test average

$\frac{2}{3}(78.4) + \frac{1}{3}X = 75$

$3\left(\frac{2}{3}\right)(78.4) + (3)\frac{1}{3}X = 75(3)$

$156.8 + X = 225$

$X = 68.2$

Ex: Greg wants to invest \$10,000 in a simple interest account. The bank he's looking into has an interest rate of 2.5%. Will he make over \$600 in interest after 2 years?
(Simple Interest Formula: I = Prt)

$P = \$10{,}000$ $\quad I = 10000 \cdot (.025) \cdot 2$

$r = .025$ $\quad I = \$500$

$t = 2$ $\quad$ NO

Ex: Cindy's take home pay monthly is \$1450, after 42% of the gross pay is deducted for taxes, savings, and benefits. How much is her gross monthly pay?

net = \$1450 42% deducted gross = ? let X = gross monthly pay

$X - .42X = 1400$

$.58X = 1450$

$X = \$2500$

Ex: Tickets to a circus are \$6 for adults, \$3 for children. If there were 4000 people total and \$17,400 was collected, how many children went?

let C = # of children 4000 people \$17,400 collected 6 adults # children

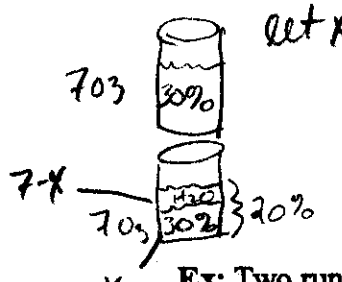
$3 \cdot C + 6(4000 - C) = \$17{,}400$

$3C + 24000 - 6C = \$17{,}400$

$-6C = -6{,}600$

$C = 2200$

Ex: (#11 in book) In a certain medical test designed to measure carbohydrate tolerance, an adult drinks 7 oz of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 oz of 20% glucose solution?



let x = amount of 30% glucose solution

$.3x = 1.4$

$x = \frac{14}{3}$ oz of 30% solution

$7 - \frac{14}{3} = \frac{7}{3}$ oz H_2O

Ex: Two runners are traveling in the same direction. The first started at 3:00 p.m. at 6 mph. The other started at 4:00 p.m. at 7 mph. How long before the second runner catches up with the first?

6mph 3p.m → 7mph 4p.m

person	rate	hours	miles
1st	6mph	t	$6t$
2nd	7mph	$t-1$	$7(t-1)$

$d = r \cdot t$

$6t = 7(t-1)$

$6t = 7t - 7$

$-t = -7$

$t = 7$ ← second person $\quad t = 7 - 1 = 6$ hours ← first person

Ex: Two women, who are 224 m apart, start walking towards each other at 1.2 m/sec and 1.8 m/sec respectively at the same instant. When will they meet, and how far will each have walked?

1.2 m/s ♀ ——— 224m ——— ♀ 1.8 m/s

person	rate	sec	miles
1st	1.2 m/s	s	d
2nd	1.8 m/s	s	$224 - d$

$d = r \cdot t$

$d = 1.2s$

$224 - d = 1.8s$

$224 - 1.2s = 1.8s$

$224 = 3s$

$s = 74.6s$

Ex: (#27 in the book) A large grain silo is to be constructed in the shape of a circular cylinder with a hemisphere attached to the top. The diameter of the silo is to be 30 ft, but the height is yet to be determined. Find the height h of the silo that will result in a capacity of $11{,}250(\pi)\text{ft}^3$.

diameter = 30' radius 15' total volume = $11{,}250\pi$ ft^3

volume of hemisphere + volume of cylinder = $11{,}250(\pi)$ ft^3

$(\frac{2}{3}\pi 15^3 \text{ft}) + (\pi 15^2 \text{ft}\, h) = 11{,}250\,\text{ft}^3$

$2500\pi\,\text{ft}^3 + h\pi 225\,\text{ft}^2 = 11{,}250\,\text{ft}^3$

$h \cdot \pi \cdot 225\,\text{ft}^2 = 9000\pi\,\text{ft}^3$

$h = 40$ ft

$40\text{ ft} + 15\text{ ft} = 55$ ft, height of silo

$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{hemisphere} = \frac{2}{3}\pi r^3$

$V_{cylinder} = \pi r^2 h$

Ex: (#30 in the book) With water from one hose, a swimming pool can be filled in 8 hours. A second, larger hose used alone can fill the pool in 5 hours. How long would it take to fill the pool if both hoses were used simultaneously?

let t = time to fill pool w/ both hoses

$\frac{1}{8}$ amount of pool filled by smaller hose in 1 hour

$\frac{1}{5}$ " " " " larger " " " "

$\frac{1}{t}$ amount of pool filled by both hoses in 1 hour

$\frac{1}{8} + \frac{1}{5} = \frac{1}{t}$

$5t + 8t = 40$

$t = 40/13$