

Algebra 2.4 Complex Numbers

$$i^2 = -1$$
$$i = \sqrt{-1}$$

Complex Number $\rightarrow a + bi$

\swarrow real part \searrow imaginary part

ex. $3+2i$, $-1-i\sqrt{3}$, 3 (or, no imaginary part), $-2i$ (or, no real #)

add $(-7-4i) + (5-2i) = -2-6i$

Mult $(-7-4i) \cdot (5-2i) = -35 - 20i + 4i + 8i^2 = -35 - 6i + 8i^2 = -35 - 6i - 8 = \boxed{-43-6i}$

$2(3-5i)^2 = 2(3-5i)(3-5i) = 2(9-15i-15i+25i^2) = 2(9-30i+25i^2) = 18-60i+50i^2$
 $= 18-60i-50 = \boxed{-32-60i}$

Cycle of i

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i \cdot i^2 = -i$

$i^4 = i^2 + i^2 = 1$

$i^5 = \sqrt{-1}$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

(divide by four use remainder)

Simplify i^{23}

$i^{23} = -i$

$4 \sqrt{23}$ 5 remainder 3

Complex Conjugates

ex $2+7i$, $2-7i$

ex $-1-i\sqrt{2}$, $-1+i\sqrt{2}$

ex $-9i$, $9i$

ex 10 , (no complex conjugate)

add $(2+7i) + (2-7i) = 4$ (adding 2 complex conjugates gives a real #)

Mult $(2+7i) \cdot (2-7i) = 4 - 49i^2 = 4 + 49 = 53$ (multiplying c.c. gives a real #)

Put in $a+bi$ format

EX $\frac{5}{2+i} \cdot \frac{2-i}{2-i} = \frac{5(2-i)}{4-i^2} = \frac{5(2-i)}{4+1} = \frac{5(2-i)}{5} = \boxed{2-i}$

Algebra 2.4 cont.

Simplify $\sqrt{-9} = \sqrt{-1 \cdot 9} = i \cdot 3 = 3i$

Simplify $-\sqrt{50} = -\sqrt{25} \sqrt{2} = -5i\sqrt{2}$

Solve using Quadratic Formula

$$x^2 + 3x + 8 = 0 \quad \frac{-3 \pm \sqrt{3^2 - 4(1)(8)}}{2(1)} = \frac{-3 \pm \sqrt{9 - 32}}{2} = \frac{-3 \pm \sqrt{-23}}{2} = \frac{-3 \pm i\sqrt{23}}{2}$$

$a=1$ $b=3$ $c=8$

Simplify $\sqrt{-9} \cdot \sqrt{-4} = 3i \cdot 2i = 6i^2 = -6$

$$1 + \overbrace{(x+7y)i}^{\text{real}} = \overbrace{x+22i}^{\text{imaginary}}$$

Combine imaginary w/ imaginary & real w/ real part
so...

$1 = x$ (since $x=1$)

$(x+7y)i = 22i$

$1 + 7y = 22$

$7y = 21$

$y = 3$