

Algebra 3.7 Operations on Functions

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$(f/g)(x) = f(x) / g(x) \quad g(x) \neq 0$$

Ex. $f(x) = 4x^3 - x + 1$ $g(x) = 3x^2 + 2$

a) $(f+g)(x) = 4x^3 - x + 1 + 3x^2 + 2 = 4x^3 + 3x^2 - x + 3$

b) $(f+g)(2) = 4(2)^3 + 3(2)^2 - 2 + 3 = 32 + 12 - 2 + 3 = 45$

c) $(f-g)(x) = 4x^3 - x + 1 - 3x^2 - 2 = 4x^3 - 3x^2 - x - 1$

d) $(f-g)(-2) = 4(-2)^3 - 3(-2)^2 - (-2) - 1 = -32 - 12 + 2 - 1 = -43$

e) $(fg)(x) = (4x^3 - x + 1)(3x^2 + 2) = 12x^5 + 5x^3 + 3x^2 - 2x + 2$

f) $(f/g)(x) = \frac{4x^3 - x + 1}{3x^2 + 2}$
 $3x^2 + 2 = 0 \quad 3x^2 = -2 \quad x^2 = -\frac{2}{3} \quad x = \pm \sqrt{-\frac{2}{3}}$ no imag so all \mathbb{R}

g) $(f/g)(1) = \frac{4(1)^3 - (1) + 1}{3(1)^2 + 2} = \frac{4 - 1 + 1}{5} = \frac{4}{5}$

Let $f(x) = 3x - 5$ $g(x) = x^2 - 4$

Find $(f/g)(x) = \frac{3x-5}{x^2-4} \quad x \neq 2, -2$

Find the domain of $(f/g)(x)$, put in interval notation $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Find the domain of $(f/g)(x)$ $f(x) = \sqrt{x+3}$ $g(x) = \sqrt{x+3}$

$$(f/g)(x) = \frac{\sqrt{x+3}}{\sqrt{x+3}}$$

$x+3 > 0 \quad x > -3 \quad (-3, \infty)$

Find the domain of $(f/g)(x)$ $f(x) = \sqrt{x+5}$ $g(x) = \sqrt{3-x}$

$$(f/g)(x) = \frac{\sqrt{x+5}}{\sqrt{3-x}} \rightarrow x+5 \geq 0 \quad x \geq -5$$

$$\sqrt{3-x} \rightarrow 3-x \geq 0 \quad -x \geq -3 \quad x \leq 3 \quad [-5, 3]$$

Q. 3.7

Find the domain of $(fg)(x)$ $f(x) = \sqrt{x+5}$ $g(x) = \sqrt{3-x}$

$$(fg)(x) = (\sqrt{x+5})(\sqrt{3-x})$$

$$\begin{aligned} &\rightarrow x+5 \geq 0 \quad x \geq -5 \\ &\quad \quad \quad \rightarrow 3-x \geq 0 \quad -x \geq -3 \quad x \leq 3 \end{aligned}$$

$$[-5, 3]$$

Find the domain of $(fg)(x)$ $f(x) = \sqrt{x+4}$ $g(x) = \frac{7}{x}$

$$(fg)(x) = (\sqrt{x+4})\left(\frac{7}{x}\right) = \frac{7\sqrt{x+4}}{x} \quad x \neq 0$$

$$[-4, 0) \cup (0, \infty)$$

Composition of Functions

$$(f \circ g)(x) = f(g(x))$$

it's read f composed of g

Ex. let $f(x) = x^2 + 2x$

$$g(x) = \sqrt{x}$$

Find $(f \circ g)(x)$

$$f(g(x)) = (\sqrt{x})^2 + 2(\sqrt{x}) = x + 2\sqrt{x}$$

$$\text{Domain} = [0, \infty)$$

Find $(g \circ f)(x)$

$$g(f(x)) = \sqrt{x^2 + 2x}$$

$$\sqrt{x(x+2)}$$

test 0, 2

$$\begin{array}{c} \begin{array}{|c|} \hline -2 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 0 \\ \hline \end{array} \end{array}$$

Domain =

$$(-\infty, -2] \cup [0, \infty)$$

Ex. Find a composite function form for y if $y = \frac{18 \cdot 2}{2 - \sqrt{x} \cdot 2}$

so let $f(x) = \frac{x}{2-x}$

$$g(x) = \sqrt{x} \cdot 2$$