Algebra 4.2 Properties of Division

Long Division Divide $3x^2-2x-7$ by $x+1$

quotient: $3x-5$

remainder: -2

$$x+1 \overline{)3x^2-2x-7}$$

$$-(3x^2+3x)$$

$$-5x-7$$

$$-(-5x-5)$$

$$-2$$

Synthetic division Divide $3x^2-2x-7$ by $x+1$

$$-1 \;|\; 3 \;\; -2 \;\; -7$$

$$\downarrow \;\; -3 \;\; 5$$

$$3 \;\; -5 \;\; -2$$

quotient: $3x-5$ rem: -2

Long Division Divide $5x^3-2x+4$ by x^2-3

(can't use synthetic division with powers greater than 2)

$$x^2+0x-3 \overline{)5x^3+0x^2-2x+4}$$

$5x$

$$-(5x^3+0x^2-15x)$$

$$13x+4$$

q: $5x$ r: $3x+4$

Remainder Theorm - If $f(x)$ is divided by $x-c$ then remainder is $f(c)$

Ex. $f(x)=3x^2-2x-7 \div x+1$ Use remainder thmn to find rem

$c=-1 \quad f(-1)=3(-1)^2-2(-1)-7=3+2-7=\underline{-2}$

Ex Use rem. thm to find the remainder when

$f(x)=2x^3-5x^2+4x+9$ & $g(x)=x-4$

$c=4 \quad f(4)=2(4)^3-5(4)^2+4(4)+9=128-80+16+9=\underline{73}$

Alg 4.2

Factor Theorm If $x-c$ is a factor of $f(x)$ then $f(c)=0$

EX Is $x-2$ a factor of $f(x)=x^3-8$?

$c=2$ $f(2)=2^3-8$ $f(2)=0$ yes

EX Is $x+5$ a factor $f(x)=3x^2-7x+25$?

$c=-5$ $f(-5)=3(-5)^2-7(-5)+25=75+35+25=125$ no

EX Find a polynomial $f(x)$ of degree 3 w/ zeros $0,-1,3$ with a leading coefficient of 2

$f(x)=2x(x+1)(x-3)$

$=2x(x^2-2x-3)=\boxed{2x^3-4x^2-6x}$

EX Find any polynomial $f(x)$ of degrees w/ zeros -4, with multiplicity of 2 and 7

$f(x)=a(x+4)^2(x-7)^7$

EX Use synthetic division to decide whether $x-3$ is a factor of x^4-2x^2+5

$x-3$ so not a factor

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -2 & 0 & 5 \\ & \downarrow & 3 & 9 & 21 & 63 \\ \hline & 1 & 3 & 7 & 21 & (68) \end{array}$$

EX Find all values of K such that $f(x)=Kx^3+x^2+K^2x+3K^2+11$ is divisible by $x+2$.

$c=-2$ $\quad 0=K(-2)^3+(-2)^2+K^2(-2)+3K^2+11$

$0=-8K+4-2K^2+3K^2+11$

$0=K^2-8K+15$

$0=(K-3)(K-5)$ $\quad (K=5,3)$