Algebra 4.4 Complex & Rational Zeros of Polynomials

EX Factor x^3-8 and find it's zeros

$$(x^3-8) = (x-2)(x^2+2x+4)$$

use quadratic formula

$$\left(\text{zeros are } 2, -1 \pm i\sqrt{3}\right)$$

$$x = \frac{-2 \pm \sqrt{4-4(1)(4)}}{2}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= -1 \pm i\sqrt{3}$$

EX Find a polynomial $f(x)$ of degree 4 that has all real coefficients, and has zeros $3+i$, $-2i$

also $3-i$, $2i$ are zeros

$$f(x) = (x-(3+i))(x-(3-i))(x+2i)(x-2i)$$
$$f(x) = (x-3-i)(x-3+i)(x^2+4)$$
$$f(x) = x^2-3x+ix-3x+9-3i-ix+3i-i^2 \ (x^2+4)$$
$$f(x) = x^2-6x+10 \ (x^2+4) \quad \checkmark \text{ stop here when i's are gone}$$

Short cut

$$(x-(a+bi))(x-(a-bi)) \qquad ^\circ\varepsilon \ (x-(3+i))(x-(3-i))$$
$$x^2-2ax+a^2+b^2 \qquad\qquad x^2-2(3)x+9+1$$
$$\qquad\qquad\qquad\qquad\qquad x^2-6x+10$$

If a polynomial $f(x)$ of degree 2 has real coefficients and $-4+3i$ is a zero, find $f(x)$

also $-4-3i$ is a zero

$$f(x) = (x-(-4+3i))(x-(-4-3i))$$
$$f(x) = x^2-2(-4)x+16+9$$
$$f(x) = x^2+8x+25$$

Alg 4.4 cont.

If polynomial $f(x)$ of degree 3 has real coefficient, and zeros $-5, 5+2i$ also $5-2i$ is a zero

$$f(x)=(x+5)(x-(5+2i))(x-(5-2i))$$
$$=(x+5)(x^2-2(5)x+25+4)$$
$$=(x+5)(x^2-10x+29)$$

<u>Rational Root Theorm</u> If $f(x)=3x^2+x-10$

Then all possible rational roots c/d,
Thus c is a factor of -10, and d is a factor of 3.
List all possible values for c/d

$c=\pm 1, \pm 2, \pm 5, \pm 10$
$d=\pm 1, \pm 3$
$c/d = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$
actual zeros : $(3x-5)(x+2)$ $\boxed{x=\frac{5}{3}, -2}$

EX. Find the zeros of $x^3-x^2-10x-8$
List all possible values of $c = \pm 1, \pm 2, \pm 4, \pm 8$
all possible values of $d = \pm 1$
all possible values of $c/d = \pm 1, \pm 2, \pm 4, \pm 8$

Try 2
```
2| 1  -1  -10  -8
  ↓    2    2  -16
   1    1   -8
```

Try -2
```
-2| 1  -1  -10  -8
   ↓   -2    6    8
    1   -3   -4    0 ✓
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$(x+2)(x^2-3x-4)$
$(x+2)(x-4)(x+1)$
$x=-2, 4, -1$

Alg 4.4 cont.

Ex Find all zeros of $x^4 + 2x^3 - 15x^2 - 14x + 56$

$c = \pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$

$d = \pm 1$

$\frac{c}{d} = \pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$

try 2 | 1   2   -15   -14   56
        ↓   2   8   -14   -56
        _____
        1   4   -7   -28   0

$(x-2)(x^3 + 4x^2 - 7x - 28)$

try -4 | 1   4   -7   -28
         ↓   -4   0   28
         _____
         1   0   -7

$(x-2)(x+4)(x^2-7)$ $\boxed{\text{zeros are } 2, -4, \sqrt{7}, -\sqrt{7}}$

$(x-2)(x+4)(x+\sqrt{7})(x-\sqrt{7})$