Algebra 5.4 Properties of Logs

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^c) = c \cdot \log_a x$$

Express in terms of x & y separately

① $\log_3(9x) = \log_3 9 + \log_3 x = 2 + \log_3 x$

② $\log_2\left(\frac{16}{y}\right) = \log_2 16 - \log_2 y = 4 - \log_2 y$

③ $\log(x^3) = 3 \cdot \log x$

④ $\log_5(x^2y^3) = \log_5 x^2 + \log_5 y^3 = 2\log_5 x + 3\log_5 y$

⑤ $\ln\left(\frac{\sqrt{x}}{\sqrt[3]{y}}\right) = \ln\sqrt{x} - \ln\sqrt[3]{y} = \ln x^{1/2} - \ln y^{1/3} = \frac{1}{2}\ln x - \frac{1}{3}\ln y$

⑥ $\ln\frac{\sqrt[3]{x^5}}{y^{\frac{3}{2}}z^{4/3}} = \ln\frac{x^{5/3}}{y^{3/2}z^{4/3}} = \ln x^{5/3} - \ln y^{3/2} - \ln z^{4/3} = \frac{5}{3}\ln x - \frac{3}{2}\ln y - \frac{4}{3}\ln z$

Common Mistakes

$\log(x+y) \neq \log x + \log y$

$\log(x-y) \neq \log x - \log y$

$\log x + \log y = \log z \quad x+y \neq z$

Write the following as one logarithm

① $\ln 10x^2y^3 - \ln xy^5 = \ln\left(\frac{10x^2y^3}{xy^5}\right) = \ln\left(\frac{10x}{y^2}\right)$

② $\ln x^2 - \ln\left(\frac{1}{y}\right)^4 - \ln(xy)^3$

$$\ln\left(\frac{x^2}{\left(\frac{1}{y}\right)^4(xy)^3}\right) = \ln\frac{x^2}{\frac{1}{y^4}x^3y^3} = \ln\left(\frac{1}{\frac{1}{y}x}\right) = \ln\left(\frac{y}{x}\right)$$

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⑤ $9\log_2 x - 5\log_2\left(\frac{1}{y}\right) - 2\log_2(xy)$

$\log_2 x^9 - \log_2\left(\frac{1}{y}\right)^5 - \log_2(xy)^2$

$\log_2 \frac{x^9}{\left(\frac{1}{y}\right)^5 (xy)^2} = \log_2 \frac{x^7}{\frac{1}{y^5}\, y^2} = \log_2 \frac{x^7}{\frac{1}{y^3}} = \boxed{\log_2 x^7 y^3}$

Solve the equation $\log(x+2) - \log x = 2\log 4$

$\log\left(\frac{x+2}{x}\right) = \log 4^2$

$\log\left(\frac{x+2}{x}\right) = \log 16$

$\frac{x+2}{x} = 16$

$x + 2 = 16x$

$2 = 15x$

$x = 2/15$

Solve for z

$2\log_3 z = 3\log_3 5$

$\log_3 z^2 = \log_3 5^3$

$\log_3 z^2 = \log_3 125$

$z^2 = 125$

$z = \pm\sqrt{125} = \pm 5\sqrt{5}$ (one of these is wrong, toss out $-5\sqrt{5}$)

$= 5\sqrt{5}$

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Solve for x $\log_2 x + \log_2(x+2) = 3$

$\log_2(x(x+2)) = 3$

$\log_2(x^2+2x) = 3$

$2^3 = x^2+2x$

$8 = x^2+2x$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x = -4, 2$ (can't be -4)

$\boxed{x=2}$

Solve for y

$\log_5(2x-5) = \log_5(10) - \log_5(2)$

$\log_5(2x-5) = \log_5(\frac{10}{2})$

$\log_5(2x-5) = \log_5(5)$

$2x-5=5$

$2x=10$

$\boxed{x=5}$

Write the following in terms of base 2

① 1 $\log_2 2 = 1$

② 3 $\log_2 8 = 3$

③ 5 $\log_2 32 = 5$

④ 0 $\log_2 1 = 0$

⑤ -4 $\log_2 \frac{1}{16} = -4$ $(2^{-4} = \frac{1}{16})$

Solve for x $\log_2 x + 3 = \log_2(4x-1)$

$\log_2 x + \log_2 8 = \log_2(4x-1)$

$\log_2(8x) = \log_2(4x-1)$

$8x = 4x-1$

$4x = -1$

$x = -\frac{1}{4}$ no solution