Algebra 5.5 Solving Exp & log Functions

Solve for x

$5^{-x} = 125$

$5^{-x} = 5^3$ ← same base

$-x = 3$

$\boxed{x = -3}$

Solve for x

$3^x = 11$ ← we can't have the same base for this problem, we must take natural log of both sides

$\ln 3^x = \ln 11$

$x \ln 3 = \ln 11$

$\boxed{x = \frac{\ln 11}{\ln 3}}$

Solve for x $2^{x-5} = 7$

$\ln 2^{x-5} = \ln 7$

$(x-5)\ln 2 = \ln 7$

$x - 5 = \frac{\ln 7}{\ln 2}$

$\boxed{x = \frac{\ln 7}{\ln 2} + 5}$

Solve for x $3^{2x} + 6(3^x) = 27$

use substitution - let $y = 3x$, and $y^2 = (3^x)^2 = 3^{2x}$

$3^{2x} + 6(3^x) = 27$

$y^2 + 6y = 27$

$y^2 + 6y - 27 = 0$

$(y+9)(y-3) = 0$

$y = -9, 3$ - we want to know what x is, not y, so plug y values in and solve.

$y = 3^x$ $\quad$ $y = 3^x$

$-9 = 3^x$ $\quad$ $3 = 3^x$

no solution $\quad$ $\boxed{x = 1}$

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Solve for x $4^x + 256 \cdot 4^{-x} = 68$

$4^x + 256\left(\frac{1}{4^x}\right) = 68$

$4^x\left(4^x + 256\left(\frac{1}{4^x}\right)\right) = 68(4^x)$ multiply both sides by 4^x to get rid of the fraction

$4^{2x} + 256 = 68(4^x)$

$4^{2x} - 68(4^x) + 256 = 0$

use substitution - let $y = 4^x$, $y^2 = 4^{2x}$

$y^2 - 68y + 256 = 0$

$(y-64)(y-4) = 0$

$y = 64, 4$ - we want to know what x so, so plug y value in, and solve

$y = 4^x$ $\quad$ $y = 4^x$

$64 = 4^x$ $\quad$ $4 = 4^x$

$x = 3$ $\quad$ $x = 1$

$\boxed{x = 1, 3}$

Solve for x $\log x^3 = (\log x)^2$ this is different from $\log x^2$

$3\log x = (\log x)^2$

$0 = (\log x)^2 - 3\log x$

$0 = \log x(\log x - 3)$ - (factored log x)

$\log x = 0$ $\quad$ $\log x - 3 = 0$

$10^0 = x$ $\quad$ $\log x = 3$

$1 = x$ $\quad$ $10^3 = x$

$x = 1000$

$\boxed{x = 1, 1000}$

Solve for x $(\log x)^4 = \log x^8$

$(\log x)^4 - \log x^8 = 0$

$(\log x)^4 - 8\log x = 0$

$\log x\left((\log x)^3 - 8\right) = 0$

$\log x = 0$ $\quad$ $(\log x)^3 - 8 = 0$

$10^0 = x$ $\quad$ $(\log x)^3 = 8$

$1 = x$ $\quad$ $\log x = 2$

$\boxed{x = 1}$ $\quad$ $10^2 = x$

$\boxed{x = 100}$

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Solve for x & approx. to 2 decimal places

$\log(x^2+4) - \log(x+2) = 2 + \log(x-2)$

$\log(x^2+4) - \log(x+2) - \log(x-2) = 2$

$\log\left(\frac{x^2+4}{(x+2)(x-2)}\right) = 2$

$\log\left(\frac{x^2+4}{x^2-4}\right) = 2$

$10^2 = \frac{x^2+4}{x^2-4}$

$100 = \frac{x^2+4}{x^2-4}$

$100(x^2-4) = x^2+4$

$100x^2 - 400 = x^2+4$

$99x^2 = 404$

$x^2 = \frac{404}{99}$

$x = \pm\sqrt{\frac{404}{99}}$

$x = \pm 2.02$ (toss out the negative value) $\boxed{x = 2.02}$

Solve for x $\quad 2^{5x+3} = 3^{2x+1}$

$\ln 2^{5x+3} = \ln 3^{2x+1}$

$(5x+3)\ln 2 = (2x+1)\ln 3$

$5x(\ln 2) + 3(\ln 2) = 2x(\ln 3) + 1(\ln 3)$

$5x(\ln 2) - 2x(\ln 3) = \ln 3 - 3(\ln 2)$ - put x's on same side

$x(5(\ln 2) - 2(\ln 3)) = \ln 3 - 3(\ln 2)$ - factor the x

$x = \frac{\ln 3 - 3(\ln 2)}{5(\ln 2) - 2(\ln 3)}$

$x = \frac{\ln 3 - \ln 2^3}{\ln 2^5 - \ln 3^2}$

$x = \frac{\ln 3 - \ln 8}{\ln 32 - \ln 9}$

$\boxed{x = \frac{\ln\left(\frac{3}{8}\right)}{\ln\left(\frac{32}{9}\right)}}$

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Change of Base Formula ★★★ not on test

$$\log_a b = \frac{\log b}{\log a} \text{ or } \frac{\ln b}{\ln a}$$

Appx $\log_2 20 = \frac{\log 20}{\log 2} = 4.32$

Appx $\frac{\log_7 64}{\log_7 4} = \frac{\frac{\log 64}{\log 7}}{\frac{\log 4}{\log 7}} = \frac{\log 64}{\log 7} \cdot \frac{\log 7}{\log 4} = \frac{\log 64}{\log 4}$

$\approx \boxed{3}$