Algebra 9.5 Systems of Linear Equations w/ More than 2 Variables
aka Solving Systems Using Matrices

Solve by elimination:

$\begin{cases} x-3y=1 \\ 2x+4y=7 \end{cases}$ (-2) $-2x+6y=-2$
$2x+4y=7$
$10y=5$
$y=1/2$

$x-3y=1$
$x-3(1/2)=1$
$x-3/2=1$
$x=5/2$

$(5/2, 1/2)$

The same problem using matrices

$\left[\begin{array}{cc|c} 1 & -3 & 1 \\ 2 & 4 & 7 \end{array}\right]$ $-2R1$ $\left[\begin{array}{cc|c} -2 & 6 & -2 \\ 2 & 4 & 7 \end{array}\right]$ $R1+R2$ $\left[\begin{array}{cc|c} -2 & 6 & -2 \\ 0 & 10 & 5 \end{array}\right]$

$10y=5$, $y=1/2$

$x-3y=1$
$x-3(1/2)=1$
$x=5/2$

$(5/2, 1/2)$

Ex $\begin{cases} x+3y-z=-3 \\ 3x-y+2z=1 \\ 2x-y+z=-1 \end{cases}$

$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 3 & -1 & 2 & 1 \\ 2 & -1 & 1 & -1 \end{array}\right]$ $-3R1+R2$ $-2R1+R3$ $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -10 & 5 & 10 \\ 0 & -7 & 3 & 5 \end{array}\right]$ $7R2$ $-10R3$

$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -70 & 35 & 70 \\ 0 & 70 & -30 & -50 \end{array}\right]$ $R2+R3$ $\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -70 & 35 & 70 \\ 0 & 0 & 5 & 20 \end{array}\right]$

goal

$5z=20$
$z=4$

$-70y+35z=70$
$-70y+35(4)=70$
$y=1$

$x+3y-z=-3$
$x+3(1)-(4)=-3$
$x=-2$

$(-2, 1, 4)$

Operations you can use on matrices

1. Multiply or divide a row by a number
2. Interchange rows (We prefer having a 1 in top left corner)
3. Add two rows together to replace a row.

Algebra 9.5 cont

EX $\begin{cases} 4x-y+3z=6 \\ -8x+3y-5z=-6 \\ 5x-4y=-9 \end{cases}$

$\left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ -8 & 3 & -5 & -6 \\ 5 & -4 & 0 & -9 \end{array}\right]$ $2R1+R2$ $\left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ 0 & 1 & 1 & 6 \\ 5 & -4 & 0 & -9 \end{array}\right]$ $-5R1$ $4R3$

$\left[\begin{array}{ccc|c} -20 & 5 & -15 & -30 \\ 0 & 1 & 1 & 6 \\ 20 & -16 & 0 & -36 \end{array}\right]$ $R1+R3$ $\left[\begin{array}{ccc|c} -20 & 5 & -15 & -30 \\ 0 & 1 & 1 & 6 \\ 0 & -11 & -15 & -66 \end{array}\right]$ $\frac{R1}{-5}$ $\left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & -11 & -15 & -66 \end{array}\right]$ $11R2+R3$

goal $\left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & -4 & 0 \end{array}\right]$

$-4z=0$ $y+z=6$ $4x-y+3z=6$

$z=0$ $y+(0)=6$ $4x-6+3(0)=6$

$y=6$ $4x=12$

$x=3$ $(3,6,0)$

EX $\begin{cases} x+3y-3z=-5 \\ 2x-y+z=-3 \\ -6x+3y-3z=4 \end{cases}$

$\left[\begin{array}{ccc|c} 1 & 3 & -3 & -5 \\ 2 & -1 & 1 & -3 \\ -6 & 3 & -3 & 4 \end{array}\right]$ $-2R1+R2$ $6R1+R3$ $\left[\begin{array}{ccc|c} 1 & 3 & -3 & -5 \\ 0 & -7 & 7 & 7 \\ 0 & 21 & -21 & -26 \end{array}\right]$ $3R2+R3$

goal $\left[\begin{array}{ccc|c} 1 & 3 & -3 & -5 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & -5 \end{array}\right]$

$0=-5$ False No solution

Algebra 9.5 cont.

EX

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & -2 & -2 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \rightleftarrows \begin{bmatrix} 1 & -2 & -2 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} \\ -2R1+R2 \\ -1R1+R3 \end{matrix} \begin{bmatrix} 1 & -2 & -2 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 3 & 3 & 0 \end{bmatrix} -3R2+5R3$$

goal $\begin{bmatrix} 1 & -2 & -2 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $0=0$

$5y+5z=0$

$y+z=0$

$y=-z$

$x-2y-2z=0$

$x-2(-z)-2(z)=0$

$x+2z-2z=0$

$x=0$

$(0,-z,z)$

True (It means 'z' is a free variable, it can be anything it wants.)

EX

$$\begin{cases} x+y=3 \\ x-z=5 \\ x+z=2 \end{cases} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 0 & -1 & 5 \\ 0 & 1 & 1 & 2 \end{bmatrix} \rightleftarrows \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & -1 & 5 \end{bmatrix} -1R1+R3 \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & 2 \end{bmatrix} R2+R3$$

goal $\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $0=4$ False (no solution)