Algebra 9.8 Determinants of nxn Matrices

1x1 Matrix

$A = [2] \qquad \det A = |A| = 2$

2x2 Matrix

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ row column $\qquad \det A = |A| = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$

(cross multiply)

Ex $A = \begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix} \qquad |A| = (-5)(2) - (4)(-3) = -10 + 12 = 2$

Ex $A = \begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix} \qquad |A| = (6)(2) - (-4)(-3) = 12 - 12 = 0$

Ex $A = \begin{bmatrix} c & d \\ -d & c \end{bmatrix} \qquad |A = (c)(c) - (d)(-d) = c^2 + d^2$

3x3 Matrix

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad |A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Algebra 9.8 cont.

EX $A = \begin{bmatrix} -5 & 4 & 1 \\ 3 & -2 & 7 \\ 2 & 0 & 6 \end{bmatrix}$

$|A| = -5\begin{vmatrix} -2 & 7 \\ 0 & 6 \end{vmatrix} - 4\begin{vmatrix} 3 & 7 \\ 2 & 6 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 2 & 0 \end{vmatrix}$

$= -5(-12-0) - 4(18-14) + 1(0+4)$

$= 60 - 16 + 4 = 48$

EX $A = \begin{bmatrix} 2 & -5 & 1 \\ -3 & 1 & 6 \\ 4 & -2 & 3 \end{bmatrix}$

$|A| = 2\begin{vmatrix} 1 & 6 \\ -2 & 3 \end{vmatrix} + 5\begin{vmatrix} -3 & 6 \\ 4 & 3 \end{vmatrix} + 1\begin{vmatrix} -3 & 1 \\ 4 & -2 \end{vmatrix}$

$= 2(3+12) + 5(-9-24) + 1(6-4)$

$= 2(15) + 5(-33) + 2 = -133$

4x4 Matrix

$A = \begin{bmatrix} 3 & -1 & 2 & 0 \\ 4 & 0 & -3 & 5 \\ 0 & 6 & 0 & 0 \\ 1 & 3 & -4 & 2 \end{bmatrix}$

$|A| = 3\begin{vmatrix} 0 & -3 & 5 \\ 6 & 0 & 0 \\ 3 & -4 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 4 & -3 & 5 \\ 0 & 0 & 0 \\ 1 & -4 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 & 5 \\ 0 & 6 & 0 \\ 1 & 3 & 2 \end{vmatrix} - 0$

$= 3\left[0\begin{vmatrix} 0 & 0 \\ 4 & 2 \end{vmatrix} + 3\begin{vmatrix} 6 & 0 \\ 3 & 2 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 3 & -4 \end{vmatrix}\right] + \left[4\begin{vmatrix} 0 & 0 \\ -4 & 2 \end{vmatrix} + 3\begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} + 5\begin{vmatrix} 0 & 0 \\ 1 & -4 \end{vmatrix}\right]$

$+ 2\left[4\begin{vmatrix} 6 & 0 \\ 3 & 2 \end{vmatrix} - 0 + 5\begin{vmatrix} 0 & 6 \\ 1 & 3 \end{vmatrix}\right]$

$= 3[0 + 3(12) + 5(-24)] + 1[0+0+0] + 2[4(12) + 5 - 6]$

$= 3[36 - 120] + 0 + 2(18)$

$= 3(-84) + 36$

$= -252 + 36$

$= -216$