Algebra 9.9 Properties of Determinants

1) If you interchange two rows in a matrix, you must change the sign of the determinant.
2) If you multiply/divide a row by a number, then you must multiply/divide the determinate by that number!
3.) If you multiply a row by a number and add it to another row, you don't change a thing!

Ex of Rule 1 $A=\begin{bmatrix}2&4&6\\1&5&0\\3&-2&5\end{bmatrix}\uparrow 2=\begin{bmatrix}1&5&0\\2&4&6\\3&-2&5\end{bmatrix}$

$\det(A)=1\begin{vmatrix}4&6\\-2&5\end{vmatrix}-5\begin{vmatrix}2&6\\3&5\end{vmatrix}+0$

$=1(20+12)-5(10-18)$

$=32-5(-8)=72$

change the sign! $\boxed{-72}$

Ex of Rule 2 $A=\begin{bmatrix}10&50&-100\\2&-3&1\\5&0&4\end{bmatrix}\div(10)\begin{bmatrix}1&5&-10\\2&-3&1\\5&0&4\end{bmatrix}$

$\det(A)=1\begin{vmatrix}-3&1\\0&-4\end{vmatrix}-5\begin{vmatrix}2&1\\5&-4\end{vmatrix}-10\begin{vmatrix}2&-3\\5&0\end{vmatrix}$

$=1(12-0)-5(-8-5)-10(0+15)$

$=12-5(-13)-150$

$=12+65-150=-73$

divide by 10! $\boxed{-\frac{73}{10}}$

Ex of Rule 3 $A=\begin{bmatrix}1&-3&5\\2&7&1\\-6&0&4\end{bmatrix}\begin{matrix}\\-2R1+R2\\6R1+R3\end{matrix}\begin{bmatrix}1&-3&5\\0&13&-9\\0&-18&34\end{bmatrix}$

$\det(A)=1\begin{vmatrix}13&-9\\-18&34\end{vmatrix}+3\begin{vmatrix}0&-9\\0&34\end{vmatrix}+5\begin{vmatrix}0&13\\0&-18\end{vmatrix}$

$=1(442-162)+3(0)+5(0)$

$=\boxed{280}$

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Cramers Rule - take a system of equations, find D, D_x, D_y, D_z

$x = \frac{|D_x|}{|D|}$ $y = \frac{|D_y|}{|D|}$ $z = \frac{|D_z|}{|D|}$

Ex Solve using Cramers Rule

$$\begin{cases} 4x + 5y = 13 \\ 3x + y = -4 \end{cases}$$

$D = \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ (left hand side of equations)

$D_x = \begin{bmatrix} 13 & 5 \\ -4 & 1 \end{bmatrix}$ (replace x column with right hand side of equation)

$D_y = \begin{bmatrix} 4 & 13 \\ 3 & -4 \end{bmatrix}$ (replace y column with left hand side of equation)

$|D| = 4(1) - 5(3) = 4 - 15 = -11$

$|D_x| = 13(1) - 5(-4) = 13 + 20 = 33$

$|D_y| = 4(-4) - 13(3) = -16 - 39 = -55$

$x = \frac{|D_x|}{|D|} = \frac{33}{-11} = -3$

$y = \frac{|D_y|}{|D|} = \frac{-55}{-11} = 5$

$(-3, 5)$

Special Cases

Ex $x = \frac{3}{0}$, $y = \frac{3}{0}$, $z = \frac{3}{0}$ → denominators of zero mean **inconsistent, no solution**

Ex $x = \frac{0}{0}$, $y = \frac{0}{0}$, $z = \frac{0}{0}$ → all values of zero mean **infinately many sol., dependent**

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Algebra 9.9

Ex Solve Using Cramers Rule

$$\begin{cases} x+3y-z=-3 \\ 3x-y+2z=1 \\ 2x-y+z=-1 \end{cases}$$

$$D=\begin{bmatrix} 1 & 3 & -1 \\ 3 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \quad D_x=\begin{bmatrix} -3 & 3 & -1 \\ 1 & -1 & 2 \\ -1 & -1 & 1 \end{bmatrix}$$

$$D_y=\begin{bmatrix} 1 & -3 & -1 \\ 3 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \quad D_z=\begin{bmatrix} 1 & 3 & -3 \\ 3 & -1 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

$$D=\begin{bmatrix} 1 & 3 & -1 \\ 3 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{matrix} \\ -3R1+R2 \\ -2R1+R3 \end{matrix} \begin{bmatrix} 1 & 3 & -1 \\ 0 & -10 & 5 \\ 0 & -7 & 3 \end{bmatrix}$$

$$|D| = 1\begin{vmatrix} -10 & 5 \\ -7 & 3 \end{vmatrix} - 3\begin{vmatrix} 0 & 5 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & -10 \\ 0 & -7 \end{vmatrix}$$
$$= 1(-30+35) = 5$$

$$D_x=\begin{bmatrix} 1 & -1 & 2 \\ -3 & 3 & -1 \\ -1 & -1 & 1 \end{bmatrix} \begin{matrix} \\ 3R1+R2 \\ R1+R3 \end{matrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 5 \\ 0 & -2 & 3 \end{bmatrix}$$

flipped R_1 & R_2

$$|D_x| = 1\begin{vmatrix} 0 & 5 \\ -2 & 3 \end{vmatrix} = 1(0+10) = 10$$

change sign! $= -10$

$$D_y=\begin{bmatrix} 1 & -3 & -1 \\ 3 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{matrix} \\ -3R1+R2 \\ -2R1+R3 \end{matrix} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 10 & 5 \\ 0 & 5 & 3 \end{bmatrix}$$

$$|D_y| = 1\begin{vmatrix} 10 & 5 \\ 5 & 3 \end{vmatrix} = 1(30-25) = 5$$

$$D_z=\begin{bmatrix} 1 & 3 & -3 \\ 3 & -1 & 1 \\ 2 & -1 & -1 \end{bmatrix} \begin{matrix} \\ -3R1+R2 \\ -2R1+R2 \end{matrix} \begin{bmatrix} 1 & 3 & -3 \\ 0 & -10 & 10 \\ 0 & -7 & 5 \end{bmatrix}$$

$$|D_z| = 1\begin{vmatrix} -10 & 10 \\ -7 & 5 \end{vmatrix} = 1(-50+70) = 20$$

$$x = \frac{|D_x|}{|D|} = \frac{-10}{5} = -2$$
$$y = \frac{|D_y|}{|D|} = \frac{5}{5} = 1$$
$$z = \frac{|D_z|}{|D|} = \frac{20}{5} = 4$$

$(-2, 1, 4)$