Luke's College Algebra Notes

Algebra 2.1 Equations 1/10/2005

Linear Equations eg $3x+2=5$ (one variable, first power)
$3x=3$ $x=1$

can have conditional - one solution $2x+2=5x-15$ $-3x=-17$ $x=\frac{17}{3}$
contradiction - no solution $2x+8=2(x-5)$ $8=-10$ F no solution $\emptyset$
identity - infinite solutions $7x+28=7(x+4)$ $7x+28=7x+28$ all real #'s $\mathbb{R}$

answer can not make the denominator $=0$

solve for x $\frac{2}{x-3}=\frac{5}{x+1}$ $2x+2=5x-15$ $17=3x$ $x=\frac{17}{3}$ ✓ since $x\neq 3$ $x\neq -1$

solve for x $\frac{1}{2x-1}=\frac{4}{8x-4}$ $\frac{1}{2x-1}=\frac{4}{4(2x-1)}$ $\frac{1}{2x-1}=\frac{1}{2x-1}$ identity $\mathbb{R}$ except $\frac{1}{2}$

solve for x $\frac{9x}{3x-1}=2+\frac{3}{x-1}$ $9x=6x-2+3$ $3x=1$ $x=\frac{1}{3}$ no solution $\emptyset$

$x\neq 3$

solve for x $\frac{2}{x-3}-\frac{5}{3-x}=-1$ $\frac{2}{x-3}-\frac{5}{-1(x-3)}=-1$ $\frac{2}{x-3}+\frac{5}{x-3}=-1$ $7=-x+3$ $x=-4$

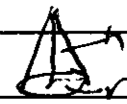
solve for x $4(8+3x)+1.7=8.5$ $32+12x+1.7=8.5$ $120x+337=85$ $x=-\frac{21}{10}$

$y\neq\pm 2$

solve for y $\frac{2}{y^2-4}-\frac{1}{y-2}=\frac{3}{y+2}$ $(y+2)(y-2)\left[\frac{2}{y^2-4}-\frac{1}{y-2}\right]=\frac{3}{y-2}(y+2)(y-2)$
$2-1(y+2)=3(y-2)$ $2-y-2=3y-6$ $6=4y$ $y=\frac{3}{2}$

Algebra 2.1 cont. 1/11/2005

Formulas



solve for h in $V = \frac{1}{3}\pi r^2 h$

$\frac{V}{\pi r^2} = \frac{1}{3}h$

$h = \frac{3V}{\pi r^2}$

solve for i $R = \frac{V}{I}$

$I = \frac{V}{R}$

solve for p $A = P + Prt$

$A = P(1+rt)$

$\frac{A}{1+rt} = P$

$P = \frac{A}{1+rt}$

solve for q $\frac{1}{f} + \frac{1}{p} = \frac{1}{q}$ $fpq\left(\frac{1}{f} + \frac{1}{p}\right) = \frac{1}{q}(fpq)$

$pq + fq = fp$

$q(p+f) = fp$

$q = \frac{fp}{p+f}$

solve for F $C = \frac{5}{9}(F-32)$

$C\frac{9}{5} = F - 32$

$\frac{9}{5}C + 32 = F$

$(x+5)^2 + 3 = (x-2)^2$

$x^2 + 10x + 25 + 3 = x^2 - 4x + 4$

$14x = -24$

$x = -\frac{12}{7}$

2.2 Applied Problems

A Simple Guideline for Problem Solving in One Variable

1. Read through the problem and make sure you understand what it is asking.
2. Draw a picture where applicable.
3. Declare a variable (x, or something) for the thing you are trying to find.
4. Set up an equation using the information you have been given (you can almost guarantee that everything that has been given must be used somewhere).
5. Solve the equation for the variable.
6. Does your answer make sense? Check your answer by plugging it back in.

Ex: A student in Calculus has test scores of 72, 84, 90, and 78. What score on the next test will give the student an average of 82?

let x = 5th test score

$\frac{72+84+90+78+x}{5} = 82$ $\quad \cancel{(5)}\left(\frac{324+x}{\cancel{5}}\right) = (82)5 \quad 324 + x = 410 \quad x = 86$

Ex: Before the final exam, Sara has test scores of 71, 79. 82, 76, and 84. If the final exam counts as 1/3 of her grade, what does she have to get in order to have a 75 average in the class?

let x = final exam grade

$\frac{71+79+82+76+84}{5} = 78.4$ test average

$\frac{2}{3}(78.4) + \frac{1}{3}x = 75$

$3(\frac{2}{3})(78.4) + (3)\frac{1}{3}x = 75(3)$

$156.8 + x = 225$

$x = 68.2$

Ex: Greg wants to invest $10,000 in a simple interest account. The bank he's looking into has an interest rate of 2.5%. Will he make over <u>$600 in interest after 2 years</u>?
(Simple Interest Formula: I = Prt)

$P = \$10{,}000$ $\quad I = 10000 \cdot (.025) \cdot 2$

$r = .025$ $\quad I = \$500$

$t = 2$ $\quad$ no

Ex: Cindy's take home pay monthly is $1450, after 42% of the gross pay is deducted for taxes, savings, and benefits. How much is her gross monthly pay?

net = $1450 42% deducted gross = ? let x = gross monthly pay

$x - .42x = 1400$

$.58x = 1450$

$x = \$2500$

Ex: Tickets to a circus are $6 for adults, $3 for children. If there were 4000 people total and $17,400 was collected, how many children went?

let c = # of children 4000 people $17,400 collected $6 adults $3 children

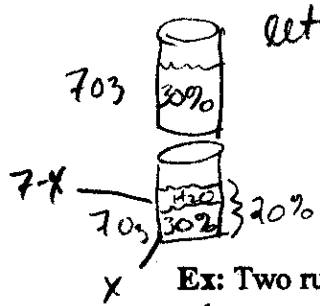
$3 \cdot c + 6(4000 - c) = \$17{,}400$

$3c + 24000 - 6c = \$17{,}400$

$-6c = -6{,}600$

$c = 2200$

Ex: (#11 in book) In a certain medical test designed to measure carbohydrate tolerance, an adult drinks 7 oz of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 oz of 20% glucose solution?



let x = amount of 30% glucose solution

$.3x = 1.4$

$x = \frac{14}{3}$ oz of 30% solution

$7 - \frac{14}{3} = \frac{7}{3}$ oz H_2O

Ex: Two runners are traveling in the same direction. The first started at 3:00 p.m. at 6 mph. The other started at 4:00 p.m. at 7 mph. How long before the second runner catches up with the first?

6mph 3pm

7mph 4pm

person	rate	hours	miles
1st	6mph	t	$6t$
2nd	7mph	$t-1$	$7(t-1)$

← $d = r \cdot t$

$6t = 7(t-1)$

$6t = 7t - 7$

$-t = -7$

$t = 7$ ← second person $t = 7 - 1 = 6$ hours – first person

Ex: Two women, who are 224 m apart, start walking towards each other at 1.2 m/sec and 1.8 m/sec respectively at the same instant. When will they meet, and how far will each have walked?

1.2 m/s   224m   1.8 m/s

person	rate	sec	miles
1st	1.2 m/s	s	d
2nd	1.8 m/s	s	224 - d

$d = r \cdot t$

$d = 1.2s$

$224 - d = 1.8s$

$224 - 1.2s = 1.8s$

$224 = 3s$

$s = 74.6s$

Ex: (#27 in the book) A large grain silo is to be constructed in the shape of a circular cylinder with a hemisphere attached to the top. The diameter of the silo is to be 30 ft, but the height is yet to be determined. Find the height *h* of the silo that will result in a capacity of 11,250(pi)ft^3.

V sphere: $\frac{4}{3}\pi r^3$
V hemisphere: $\frac{2}{3}\pi r^3$
V cylinder: $\pi r^2 h$

diameter = 30′ radius 15′ total volume = $11{,}250\pi$ ft³

volume of hemisphere + volume of cylinder = $11{,}250(\pi)$ ft³

$(\frac{2}{3}\pi 15^3 ft) + (\pi 15^2 ft\, h) = 11{,}250 ft^3$

$2500\pi ft^3 + h\pi 225 ft^2 = 11{,}250 ft^3$

$h\pi \cdot 225 ft^2 = 9000\pi ft^3$

$h = 40$ ft

40 ft + 15 ft = 55 ft, height of silo

Ex: (#30 in the book) With water from one hose, a swimming pool can be filled in 8 hours. A second, larger hose used alone can fill the pool in 5 hours. How long would it take to fill the pool if both hoses were used simultaneously?

let t = time to fill pool w/ both hoses

$\frac{1}{8}$ amount of pool filled by smaller hose in 1 hour

$\frac{1}{5}$ " " " " larger " " " "

$\frac{1}{t}$ amount of pool filled by both hoses in 1 hour

$\frac{1}{8} + \frac{1}{5} = \frac{1}{t}$

$5t + 8t = 40$

$t = 40/13$

Algebra 2.3 Quadratic Equations 1/13/2005

standard form $ax^2+bx+c=0$ $a\neq 0$

zero factor theorem if $a\cdot b=0$ then either $a=0$ or $b=0$

Ex: Solve $x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6,-1$

ex. $16x^2-9$

$16x^2-9=0$

$(4x+3)(4x-3)=0$

$x=-3/4, 3/4$

Solve: $2x(4x+15)=27$

$8x^2+30x-27=0$

$(2x+9)(4x-3)=0$

$x=-9/2, 3/4$

solve $x(x-5)=0$

$x=0,5$

solve $\frac{2x}{x+3}+\frac{5}{x}=4+\frac{18}{x^2+3x}$ $\boxed{x\neq 0,-3}$

$x(x+3)\left(\frac{2x}{x+3}+\frac{5}{x}\right)=\left(4+\frac{18}{x^2+3x}\right)x(x+3)$

$2x^2+5x+15=4x^2+12x+18$

$-2x^2-7x-3=0$

$2x^2+7x+3=0$

$(2x+1)(x+3)=0$

$x=-1/2, \cancel{-3}$ $x=-1/2$

Algebra 2.3 cont.

Completing the Square

1) x^2+6x ___ $\qquad 6(\frac{1}{2})=3 \quad 3^2=9$

$x^2+6x+9 \qquad (x+3)^2$

2) x^2-10x ___ $\qquad 10(\frac{1}{2})=5 \quad 5^2=25$

$x^2-10x+25 \qquad (x-5)^2$

3) x^2-3x ___ $\qquad 3(\frac{1}{2})=\frac{3}{2} \quad (\frac{3}{2})^2=\frac{9}{4}$

$x^2-3x+\frac{9}{4} \qquad (x-\frac{3}{2})^2$

Solve by completing the square

1. $x^2+8x-11=0$

$x^2+8x=11 \qquad 8(\frac{1}{2})=4 \quad 4^2=16$

$x^2+8x+16=11+16$

$(x+4)^2=27$

$\sqrt{(x+4)^2}=\pm\sqrt{27}$

$x+4=\pm 3\sqrt{3}$

$x=4\pm 3\sqrt{3}$

2. $4x^2-12x-11=0$ (Note: coefficent of x^2 must be 1!)

$x^2-3x-\frac{11}{4}=0$

$x^2-3x=\frac{11}{4} \qquad 3(\frac{1}{2})=\frac{3}{2} \quad (\frac{3}{2})^2=\frac{9}{4}$

$x^2-3x+\frac{9}{4}=\frac{11}{4}+\frac{9}{4}$

$(x-\frac{3}{2})^2=5$

$\sqrt{(x-\frac{3}{2})^2}=\pm\sqrt{5}$

$x-\frac{3}{2}=\pm\sqrt{5}$

$x=\frac{3}{2}\pm\sqrt{5}$

Algebra 2.3 cont.

Quadratic Formula $\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Solve using the Quad Formula

1.) $6x^2 - 2 = x$ $\quad x = \frac{--1 \pm \sqrt{-1^2 - 4(6)(-2)}}{2(6)} = \frac{1 \pm \sqrt{1+48}}{12} = \frac{1 \pm \sqrt{49}}{12}$

$6x^2 - x - 2 = 0$ $\quad x = \frac{1 \pm 7}{12} = \frac{8}{12}, -\frac{6}{12} = \frac{2}{3}, -\frac{1}{2}$

$a=6 \quad b=-1 \quad c=-2$

2.) $\frac{3}{2}y^2 - 4y - 1 = 0$ $\quad y = \frac{--8 \pm \sqrt{-8^2 - 4(3)(-2)}}{2(3)} = \frac{8 \pm \sqrt{64+24}}{6} = \frac{8 \pm \sqrt{88}}{6}$

$3y^2 - 8y - 2 = 0$ $\quad \frac{8 \pm 2\sqrt{22}}{6} = \frac{4 \pm \sqrt{22}}{3}$

$a=3 \quad b=-8 \quad c=-2$

3) $\frac{5x}{x^2+9} = -1$ $\quad x = \frac{-5 \pm \sqrt{5^2 - 4(1)(9)}}{2(1)} = \frac{-5 \pm \sqrt{25-36}}{2} = \frac{-5 \pm \sqrt{-11}}{2}$

$(x^2+9)\frac{5x}{x^2+9} = -1(x^2+9)$

$5x = -x^2 - 9$ $\quad$ for sec 2.3 stop here

$x^2 + 5x + 9 = 0$

$a=1 \quad b=5 \quad c=9$

$100x^2 - 220x - 879 = 0$ (iLrn problem - solve by completing the square)

$\frac{100x^2}{100} - \frac{220x}{100} - \frac{879}{100} = 0$

$x^2 - \frac{11}{5}x = \frac{879}{100}$ $\quad \frac{11}{5}\left(\frac{1}{2}\right) = \frac{11}{10} \quad \left(\frac{11}{10}\right)^2 = \frac{121}{100}$

$x^2 - \frac{11}{5}x + \frac{121}{100} = \frac{879}{100} + \frac{121}{100}$

$(x - \frac{11}{10})^2 = \frac{1000}{100}$

$\sqrt{(x-\frac{11}{10})^2} = \pm\sqrt{10}$

$(x - \frac{11}{10}) = \pm\sqrt{10}$

$x = \frac{11}{10} \pm \sqrt{10}$

Algebra 2.4 Complex Numbers

$i^2 = -1$
$i = \sqrt{-1}$

Complex Number $\to$ $a + bi$

real part imaginary part

ex. $3+2i$, $-1-i\sqrt{3}$, 3 (ok, no imaginary part), $-2i$ (ok, no real #)

add $(-7-4i)+(5-2i) = -2-6i$

Mult $(-7-4i)\cdot(5-2i) = -35-20i+14i+8i^2 = -35-6i+8i^2 = -35-6i-8 = -43-6i$

$2(3-5i)^2 = 2(3-5i)(3-5i) = 2(9-15i-15i+25i^2) = 2(9-30i+25i^2) = 18-60i+50i^2$
$= 18-60i-50 = -32-60i$

Cycle of i

$i = \sqrt{-1}$ $\qquad$ $i^5 = \sqrt{-1}$ $\qquad$ (divide by four use remainder)

$i^2 = -1$ $\qquad$ $i^6 = -1$

$i^3 = i \cdot i^2 = -i$ $\qquad$ $i^7 = -i$ $\qquad$ Simplify i^{23} $\qquad$ $4\overline{)23}$ 5 remainder 3

$i^4 = i^2 + i^2 = 1$ $\qquad$ $i^8 = 1$ $\qquad$ $i^{23} = -i$

Complex Conjugates

EX $2+7i$, $2-7i$

EX $-1-i\sqrt{2}$, $-1+i\sqrt{2}$

EX $-9i$, $9i$

EX 10, (no complex conjugate)

add $(2+7i)+(2-7i) = 4$ (adding 2 complex conjugates gives a real #)

Mult $(2+7i)\cdot(2-7i) = 4-49i^2 = 4+49 = 53$ (multiplying c.c. gives a real #)

Put in $a+bi$ format

EX $\frac{5}{2+i}$ $\qquad$ $\frac{5}{2+i} \cdot \frac{5}{2-i} = \frac{5(2-i)}{4-i^2} = \frac{5(2-i)}{4+1} = \frac{5(2-i)}{5} = 2-i$

Algebra 2.4 cont.

Simplify $\sqrt{-9} = \sqrt{-1} \cdot \sqrt{9} = i \cdot 3 = 3i$

Simplify $-\sqrt{-50} = -\sqrt{-25}\,\sqrt{2} = -5i\sqrt{2}$

Solve using Quadratic Formula

$x^2 + 3x + 8 = 0 \qquad \frac{-3 \pm \sqrt{3^2 - 4(1)(8)}}{2(1)} = \frac{-3 \pm \sqrt{9-32}}{2} = \frac{-3 \pm \sqrt{-23}}{2} = \frac{-3 \pm i\sqrt{23}}{2}$

$a=1 \quad b=3 \quad c=8$

Simplify $\sqrt{-9} \cdot \sqrt{-4} = 3i \cdot 2i = 6i^2 = -6$

real

$1 + (x+7y)i = x + 22i$

imaginary

combine imaginary w/ imaginary & real w/ real part

so...

$1 = x \qquad (\text{since } x = 1)$

$(x+7y)i = 22i$

$1 + 7y = 22$

$7y = 21$

$y = 3$

Algebra 2.5 Other Types of Equations

Absolute Value Equations

EX. $|x| = 4$ $x = 4, -4$

EX. $2|x-4| - 12 = 0$

$2|x-4| = 12$

$|x-4| = 6$

$x-4=6$ $x-4=-6$

$x=10$ $x=-2$

EX $|3x-8| = -3$ Can't have a negative w/ absolute value, no solution $\emptyset$

Grouping

EX. $7x^3 - 14x^2 - 5x + 10 = 0$

$7x^2(x-2) - 5(x-2) = 0$

$(7x^2-5)(x-2) = 0$

$x = \pm\sqrt{\frac{5}{7}} = \pm\frac{\sqrt{35}}{7}$ $x=2$

Rational Exponents

EX $y^{3/2} = 5y$

$y^{3/2} - 5y = 0$

$y(y^{1/2} - 5) = 0$

$y=0$ $y^{1/2} - 5 = 0$ $y = 25$

EX $y^{3/4} = 16$

$(y^{3/4})^{4/3} = (16)^{4/3}$

$y^{12/12} = (\sqrt[3]{16})^4$

$y = (\sqrt[3]{16})^4$

EX $x^{3/2} = 27$

$(x^{3/2})^{2/3} = (27)^{2/3}$ (raise both sides by the reciprical of the value)

$x^{6/6} = (\sqrt[3]{27})^2$

$x = 3^2$

$x = 9$

Algebra 2.5

Radical Equations

$\sqrt{7-x} = x-5$

$7-x = (x-5)^2$

$7-x = x^2-10x+25$

$0 = x^2-9x+18$

$0 = (x-6)(x-3)$

$x = 6, 3$ must check answers

$\sqrt{7-6} = 6-5$

$\sqrt{1} = 1$ ✓

$\sqrt{7-3} = 3-5$

$\sqrt{4} = -2$

$2 \neq -2$

$x = 6$

$\sqrt[3]{7x-4} - 2 = 0$

$\sqrt[3]{7x-4} = 2$

$7x-4 = 8$

$7x = 12$

$x = \frac{12}{7}$

$\sqrt[3]{7(\frac{12}{7})-4} - 2 = 0$

$\sqrt[3]{12-4} - 2 = 0$

$\sqrt[3]{8} - 2 = 0$

$2 - 2 = 0$ ✓

Quadratic Like Equations

EX. $x^4 - 25x^2 + 144 = 0$

$(x^2-16)(x^2-9) = 0$

$(x+4)(x-4)(x+3)(x-3) = 0$ $\quad x = 4, -4, 3, -3$

EX $6w - 23w^{1/2} + 20 = 0$ $\quad$ Let $w^{1/2} = x$ $\quad x^2 = w$

$6x^2 - 23x + 20 = 0$

$(3x-4)(2x-5) = 0$

$3x = 4 \quad 2x = 5$

$x = 4/3 \quad x = 5/2 \Rightarrow w = \frac{16}{9}, \frac{25}{4}$

Algebra 2.5

Difference of Two Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of Two Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

EX Factor $8x^3 - 27$

$a = 2x \quad b = 3$

$(2x-3)(4x^2 + 6x + 9)$

Algebra 2.6 Inequalities

Write the following in interval notation

1. $x > 5$ $(5, \infty)$
2. $x \le 3$ $(-\infty, 3]$
3. $0 < x \le 7$ $(0, 7]$
4. $4 < x$ $(4, \infty)$ same as $x > 4$
5. $5 \ge x > -1$ $(-1, 5]$

Write the following as an equality

1. $[-1, \infty)$ $x \ge -1$
2. $(5, 7)$ $5 < x < 7$
3. $(-\infty, 0)$ $x < 0$
4. $[-1, 8)$ $-1 \le x < 8$

Solve $-5x + 2 \le -3x + 8$

$-2x \le 6$ //when dividing by a neg. #, switch signs

$x \ge -3$ $[-3, \infty)$

Solve $-9 \le \frac{1}{2}x - 3 < 5$

$-6 \le \frac{1}{2}x < 8$

$-12 \le x < 16$ $[-12, 16)$

Solve $\frac{7}{x-3} > 0$ when is it positive?

$x - 3 > 0$

$x > 3$ $(3, \infty)$

Solve $\frac{2}{5-x} < 0$ when is it negative

$5 - x < 0$

$-x < -5$ $x > 5$ $(5, \infty)$

Solve $\frac{-5}{2x+7} > 0$ denominator must be negative to make answer positive

$2x + 7 < 0$

$x < -\frac{7}{2}$ $\left(-\infty, -\frac{7}{2}\right)$

Algebra 2.6 Inequalities cont.

Absolute Values in Inequalities

General Rules

$|x| < a$ then $-a < x < a$

$|x| > a$ then $x < a$ or $x > a$

solve $|x| \geq 5$ $\quad x \leq -5 \quad x \geq 5 \quad (-\infty, -5] \cup [5, \infty)$

solve $|x| < 12$ $\quad -12 < x < 12 \quad (-12, 12)$

solve $|x| < -3$ $\quad$ no solution $\emptyset$

solve $|x| > -4$ $\quad (-\infty, \infty)$

solve $|6x-1| < 8$

$-8 < 6x-1 < 8$

$-\frac{7}{6} < x < \frac{9}{6} \quad (-\frac{7}{6}, \frac{3}{2})$

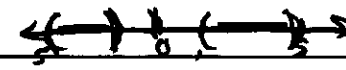
solve $\left|\frac{4x-9}{2}\right| \geq 11$ $\quad \frac{4x-9}{2} \leq -11 \quad \frac{4x-9}{2} \geq 11 \quad -\frac{13}{4} \geq x \geq \frac{31}{4}$

$(-\infty, -\frac{13}{4}] \cup [\frac{31}{4}, \infty)$

$4x-9 \leq -22 \quad 4x-9 \geq 22$

$x \leq -\frac{13}{4} \quad x \geq \frac{31}{4}$

solve $1 < |x| < 5$ abs value wedged between 2#'s

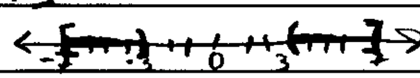


$|x| > 1 \quad |x| < 5 \quad (-5, -1) \cup (1, 5)$

solve $-2 \leq |x| \leq 4$ same as $0 \leq |x| \leq 4$

$[-4, 4]$

solve $3 < |x| \leq 7$



$[-7, 3) \cup (3, 7]$

solve $C = \frac{5}{9}(F-32)$ What values of F correspond to the values of C, such that $30 \leq C \leq 40$?

$30 \leq \frac{5}{9}(F-32) \leq 40$

$(\frac{9}{5})30 \leq \frac{5}{9}(\frac{9}{5})(F-32) \leq 40(\frac{9}{5})$

$54 \leq F-32 \leq 72$

$86 \leq F \leq 104 \quad [86, 104]$

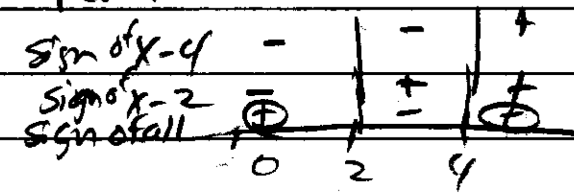
Algebra 2.7 More on Inequalities

Inequalities of degree 2 or more

Solve $x^2-6x+8>0$ ← needs to be positive

$(x-4)(x-2)>0$

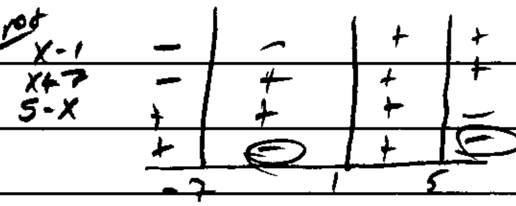
$x=4,2$ are test values



$(-\infty,2)\cup(4,\infty)$

Solve $(x-1)(x+7)(5-x)\le 0$ ← want neg

$x=1,-7,5$ (test values)



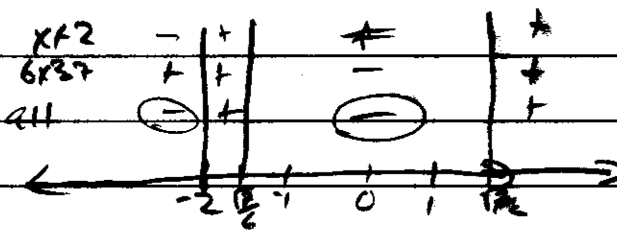
$[-7,1]\cup[5,\infty)$

Solve $6x^3+12x^2-7x-14\le 0$ ← want neg

$6x^2(x+2)-7(x+2)\le 0$

$(x+2)(6x^2-7)\le 0$

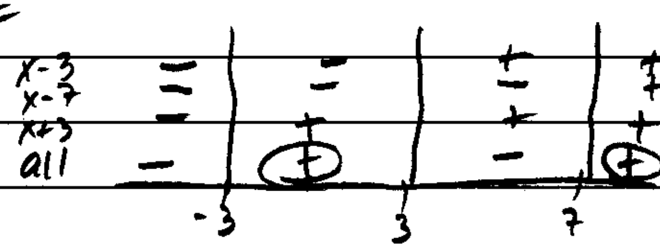
$x=-2,\ \pm\sqrt{\frac{7}{6}}$



$(-\infty,-2]\cup[-\sqrt{\frac{7}{6}},\sqrt{\frac{7}{6}}]$

Solve $\dfrac{x-3}{x^2-4x-21}\ge 0$ pos #

$\dfrac{x-3}{(x+3)(x-7)}\ge 0$



test values $3, 7, -3$, but $x\ne -3,7$ (use paren/not brackets)

$(-3,3]\cup(7,\infty)$

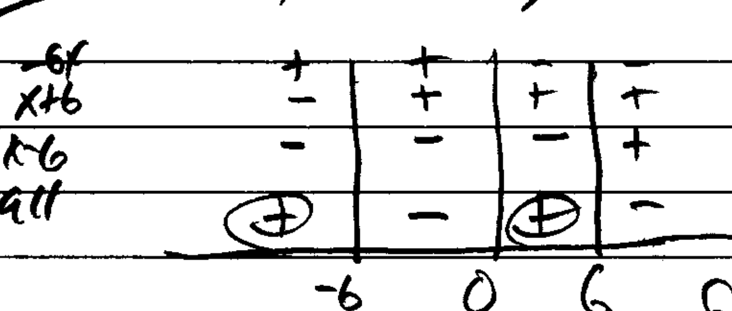
Solve $\dfrac{-6x}{x^2-36}>0$ pos #

$\dfrac{-6x}{(x+6)(x-6)}$

test values

$x=0,-6,6$

$x\ne 6,-6$



$(-\infty,-6)\cup(0,6)$

Solve

$$\frac{(x+4)^2(5-x)}{(x+3)^2(x-3)} \le 0$$ want neg #

$x \ne -3, 3$

test values $-4, 5, -3, 3$

		-4		-3		3		5	
$(x+4)^2$	+		+		+		+		+
$(5-x)$	+		+		+		+		−
$(x+3)^2$	+		+		+		+		+
$(x-3)$	−		−		−		+		+
all	−		−		−		+		−

~~$(-\infty, 3) \cup [5, \infty)$~~

$(-\infty, -3) \cup (-3, 3) \cup [5, \infty)$

Summary of Inequalities

Absolute Value Inequalities

Isolate the absolute value first, and then follow either step one or two:

1. Less than symbol: less th<u>and</u>, set up an "and" compound inequality and solve.
2. Greater than symbol: great<u>or</u> than, set up an "or" compound inequality and solve.

Ex 1: Solve $|3x-2|-7<0$

$|3x-2|<7$

$-7<3x-2<7$

$-5<3x<9$

$-5/3<x<3$

$(-\frac{5}{3},3)$

Ex 2: Solve $|-2x+9|\geq 1$

$-2x+9\leq -1$ or $-2x+9\geq 1$

$-2x\leq -10$ $\quad$ $-2x\geq -8$

$x\geq 5$ or $x\leq 4$

$(-\infty,4]\cup[5,\infty)$

Ex 3: Solve $|6x+8|<-1$

no solution $\emptyset$

Ex 4: Solve $|6x+8|>-1$

same as $|6x+8|\geq 0$

$\mathbb{R}$ or $(-\infty,\infty)$

Linear Inequalities (highest power of x is 1)

Get x by itself on the left hand side. Your answer should be a single interval.

Ex 5: Solve $-3x+2>x+10$

$-4x>8$

$x<-2$

$(-\infty,-2)$

Ex 6: Solve $(2x-3)(5x+1)\leq 10x^2-x$

$10x^2+2x-15x-3\leq 10x^2-x$

$-13x-3\leq -x$

$-12x-3\leq 0$

$-12x\leq 3$

$x\geq -3/12$

$x\geq -1/4$ $\quad$ $[-1/4,\infty)$

Inequalities of degree 2 or more

Get all terms on the left side (zero on the right), factor the left side, and set up a sign diagram! This is the only type of inequality you use a sign diagram for. You must be careful not to include values that make any denominators zero, and be sure to include values that make the numerator zero when you have $\leq$, or $\geq$.

Ex 7: Solve $x^2+12x \ge -5x+60$

$x^2+17x-60 \ge 0$

$(x+20)(x-3) \ge 0$

test values are $-20, 3$

		-20		3	
$x+20$	$-$		$+$		$+$
$x-3$	$-$		$-$		$+$
all	$+$		$-$		$+$

$(-\infty,-20]\cup[3,\infty)$

Ex 8: Solve $-3x^2(x+2)(7-x)\le 0$

test values are $0,-2,7$

		-2		0		7	
$-3x^2$	$-$		$-$		$-$		$-$
$x+2$	$-$		$+$		$+$		$+$
$7-x$	$+$		$+$		$+$		$-$
all	$+$		$-$		$-$		$+$

$[-2,7]$

$x\ne -7,4$

$x=3,2$

Ex 9: Solve $\dfrac{(x-3)(x^2-5x+6)}{x^2+3x-28}\ge 0$

$$\frac{(x-3)(x-3)(x-2)}{(x+7)(x-4)}\ge 0$$

test values are $-7,2,3,4$

		-7		2		3		4	
$(x-3)^2$	$+$		$+$		$+$		$+$		$+$
$x-2$	$-$		$-$		$+$		$+$		$+$
$x+7$	$-$		$+$		$+$		$+$		$+$
$x-4$	$-$		$-$		$-$		$-$		$+$
all	$-$		$+$		$-$		$-$		$+$

$(-7,2]\cup\{3\}\cup(4,\infty)$

Ex 10: Solve $\dfrac{(x-3)^2(x-2)}{(x+7)(x-4)}>0$

$x\ne -7,4$

same as last example

$(-7,2)\cup(4,\infty)$

$x\ne 2$

$x=3,4$

Ex 11: Solve $\dfrac{-(x+4)(3-x)}{(x-2)^2}\le 0$

$$\frac{(x-3)(x+4)}{(x-2)^2}\le 0$$

test values $2,3,-4$

		-4		2		3	
$(x-3)$	$-$		$-$		$-$		$+$
$(x+4)$	$-$		$+$		$+$		$+$
$(x-2)^2$	$+$		$+$		$+$		$+$
all	$+$		$-$		$-$		$+$

$-4 \to 3$

$[-4,2)\cup(2,3]$

Ex 12: Solve $x^2(x+2)(7-x)\le 0$

test values $0,-2,7$

		-2		0		7	
x^2	$+$		$+$		$+$		$+$
$x+2$	$-$		$+$		$+$		$+$
$7-x$	$+$		$+$		$+$		$-$
all	$-$		$+$		$+$		$-$

$(-\infty,-2]\cup\{0\}\cup[7,\infty)$

Due thurs

Bonus (2pts): The number of miles M that a certain compact car can travel on 1 gallon of gasoline is related to its speed v (in mi/hr) by: $M=-\frac{1}{30}v^2+\frac{5}{2}v$, for $0<v<70$. For what speeds will M be at least 45?

Luke Spence

x2

Bonus (Summary of Inequalities)

The number of miles M that a certain compact car can travel on 1 gallon of gasoline is relative to it's speed v in mph by $M = -\frac{1}{30}v^2 + \frac{5}{2}v$, for $0 < v < 70$
For what speed will M be at least 45?

$$-\frac{1}{30}v^2 + \frac{5}{2}v = M$$

$$-\frac{1}{30}v^2 + \frac{5}{2}v \leq 45$$

$$30\left(-\frac{1}{30}v^2 + \frac{5}{2}v\right) \leq 45\,(30)$$

$$-v^2 + 75v \leq 1350$$

$$-v^2 + 75v - 1350 \leq 0$$

$$v^2 - 75v + 1350 \geq 0$$

$$(v-30)(v-45) \geq 0$$

test values 30, 45

$(v-30)$	−	+	+
$(v-45)$	−	−	+
all	+	−	+
	30	45	

[30, 45]

Algebra 3.1 Distance & Midpoint Formula

Distance Formula $\boxed{d(P_1, P_2) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$

Ex. Find the distance between $(-2,5)$ & $(0,7)$

$d = \sqrt{(0-{-2})^2+(7-5)^2} = \sqrt{4+4} = 2\sqrt{2}$

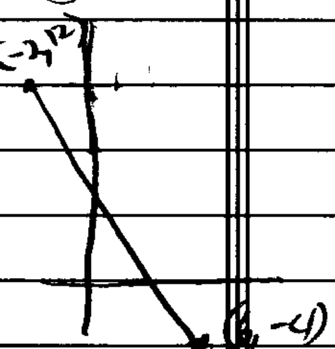
Ex. Find the distance between $(-11,4)$ & $(3,-3)$

$d = \sqrt{(3-{-11})^2+(-3-4)^2} = \sqrt{14^2+(-7)^2} = \sqrt{196+49} = 7\sqrt{5}$

Midpoint Formula $\boxed{M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)}$

Ex. Find the midpoint between $(-4,-5)$ & $(2,-1)$

$M = \left(\frac{-4+2}{2}, \frac{-5+-1}{2}\right) = \left(\frac{-2}{2}, \frac{-6}{2}\right) = (-1,-3)$



Ex Given $A(6,-4)$ & $B(-2,12)$ Find the point on the segment AB that is 3/4 of the way from A to B.

x coordinate $x_1 + \frac{3}{4}(x_2-x_1) = 6+\frac{3}{4}(-2-6) = 6+\frac{3}{4}(-8) = 6-6 = 0$

y coordinate $y_1 + \frac{3}{4}(y_2-y_1) = -4+\frac{3}{4}(12-{-4}) = -4+\frac{3}{4}(16) = -4+12 = 8$

$(0,8)$

Ex Find a formula that expresses the fact that $P(x,y)$ is 3 units away from the origin.

$\sqrt{(x-0)^2+(y-0)^2} = 3$

$\sqrt{x^2+y^2} = 3$

$x^2+y^2 = 9$

Algebra 3.1

EX. Find all points on the y-axis that are 5 units away from (3,4)

(0, y)

$$\sqrt{(0-3)^2+(y-4)^2}=5$$

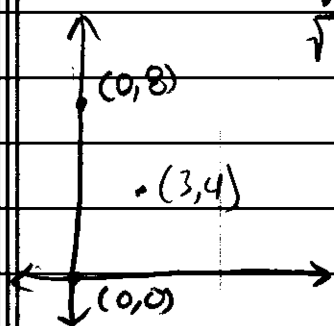
$$\sqrt{(-3)^2+(y-4)^2}=5$$

$$9+(y-4)^2=25$$

$$(y-4)^2=16$$

$$y-4=\pm 4$$

$$y=0,8 \qquad (0,0),(0,8)$$



EX Find all points on the x-axis that are 7 units away from (1,-3)

(X, 0)

$$7=\sqrt{(x-1)^2+(0-3)^2}$$

$$49=(x-1)^2+9$$

$$40=(x-1)^2$$

$$\pm\sqrt{40}=x-1$$

$$x=1\pm 2\sqrt{10} \qquad (1+2\sqrt{10},0),(1-2\sqrt{10},0)$$

EX Given $P_1(-5,1)$ find P_2 such that (3,-2) is the midpoint of P_1 & P_2

$$M=\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}$$

$$m_x=\frac{x_1+x_2}{2} \qquad m_y=\frac{y_1+y_2}{2}$$

$$3=\frac{-5+x_2}{2} \qquad -2=\frac{1+y_2}{2}$$

$$6=-5+x_2 \qquad -4=1+y_2$$

$$x_2=11 \qquad y_2=-5$$

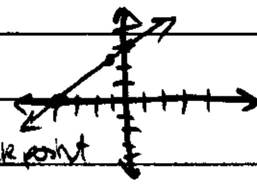
(11, -5)

Algebra 3.2 Graphs of Equations (Circles)

Graph using intercepts

Ex Graph $-3x+4y=12$

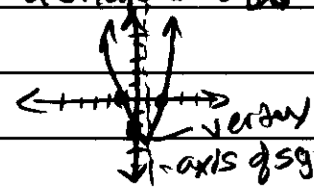
x	y
-4	0
0	3
-1	9/4 - check point



Quadratic Equations (Degree of 2 or more - U shape ∪, ∩)

Ex Graph $y=x^2-x-2$

x	y
0	-2
2, -1	0



Circles

General Equation $\boxed{(x-h)^2+(y-k)^2=r^2}$

Ex Find the center and radius of $(x+3)^2+(y-5)^2=25$

center $(-3,5)$ radius $=\sqrt{25}=5$

Ex Find the center and radius of $(x+\frac{2}{3})^2+(y+\frac{7}{3})^2=18$

center $(-\frac{2}{3},-\frac{7}{3})$ radius $=\sqrt{18}=3\sqrt{2}$

Ex Write an equation for a circle w/ center $(1,-6)$ & radius of 10

$(x-1)^2+(y+6)^2=100$

Ex Write an equation for a circle w/ center $(-4,0)$ & radius of $5\sqrt{3}$

$(x+4)^2+y^2=75$

Ex Find ~~the equation~~ of a circle w/ endpoints of diameter $(-2,5)$ & $(4,5)$

$d=\sqrt{(4--2)^2+(5-5)^2}=\sqrt{36}=6$ radius $=6/2=3$

$M=\left(\frac{-2+4}{2},\frac{5+5}{2}\right)=(1,5)$

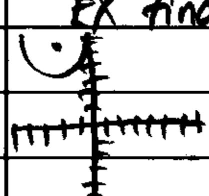
$(x-1)^2+(y-5)^2=9$

Ex Find the equation of a circle w/ center $(-3,5)$ that is tangent to the x-axis

r=5 

$(x+3)^2+(y-5)^2=25$

Ex Find the equation of a circle w/ center $(-2,7)$ that's tangent to the y-axis

r=2 

$(x+2)^2+(y-7)^2=4$

Algebra 3.2

EX Find the center & radius of the following $x^2-6x+y^2+4y-7=0$

$$x^2-6x+\underline{9}+y^2+4y+\underline{4}=7+9+4$$

$$(x+3)^2+(y+2)^2=20$$

center $(-3,-2)$ radius $=\sqrt{20}=2\sqrt{5}$

EX Find the center & radius of the following $x^2+10x+y^2-3y+2=0$

$$x^2+10x+\underline{25}+y^2-3y\underline{+\tfrac{9}{4}}=-2+25+\tfrac{9}{4}$$

$$(x+5)^2+(y-\tfrac{3}{2})^2=\frac{-8+100+9}{4}=\frac{101}{4}$$

center $(-5,\tfrac{3}{2})$ radius $=\sqrt{\tfrac{101}{4}}=\frac{\sqrt{101}}{2}$

EX Is the point $(3,5)$ inside, outside, or on the circle

$$(x-2)^2+(y-1)^2=36$$

$$(3-2)^2+(5-1)^2=36$$

$$1+16=36$$

$17<36$ less than means inside the circle

Test for Symmetry

Symmetry with respect to y-axis

EX $y=3x^2$ (plug in $-x$, answer should be identicle)

$$y=3(-x)^2$$

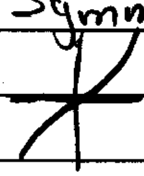
$y=3x^2$ Symmetrical w/ y axis

EX $y=5x^3-x$

$$y=5(-x)^3--x$$

$y=-5x^3+x$ not symmetrical w/ y axis

Symmetry with respect to the origin



(x,y)

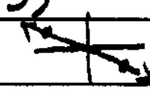
$(-x,-y)$ Plug in both x & y

Algebra 3.3 Lines

Slope
$m = \frac{y_2 - y_1}{x_2 - x_1}$

EX Find the slope between $(-6,2)$ & $(5,-3)$

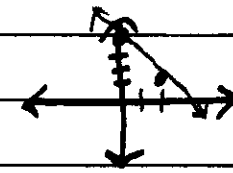
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-2}{5--6} = \frac{-5}{11}$ ← down 5, ← right 11



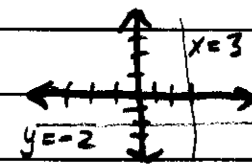
Slope Inter
$y = mx + b$
m = slope
b = y int

EX Let's graph the equation $y = -\frac{3}{2}x + 4$

$m = -\frac{3}{2}$ $\quad b = 4$



EX Graph the equation $x = 3$ & $y = -2$ on the same axis



Point Slope
$y - y_1 = m(x - x_1)$

What is the slope of $x = 3$? Undefined

$(3,4), (3,0) = m = \frac{4-0}{3-3} = \frac{4}{0}$ so undefined

What is the slope of $y = -2$ $\quad m = 0$

$(-3,-2), (0,-2) = m = \frac{-2--2}{0--3} = \frac{0}{3} = 0$

Standard Form
$Ax + By = C$
(A pos, no frac)

EX Write an equation for a line with a slope $m = -2$ through $(0,-5)$

$m = -2 \quad b = -5 \quad y = mx + b \quad y = -2x - 5$

EX Write an equation for a line w/ slope $\frac{1}{2}$, through $(6,-1)$

$m = \frac{1}{2} \quad y = \frac{1}{2}x + b \quad -1 = \frac{1}{2}(6) + b \quad -1 = 3 + b \quad b = -4 \quad y = \frac{1}{2}x - 4$

another way: $y - y_1 = m(x - x_1) \quad y + 1 = m(x - 6) \quad y = \frac{1}{2}x - 4$

EX Write an equation for the line between $(-7,-2)$ & $(4,-1)$

$m = \frac{-1+2}{4+7} = \frac{1}{11}$

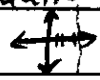
$y = \frac{1}{11}x + b \quad -1 = \frac{1}{11}(4) + b \quad b = -1 - \frac{4}{11} = \frac{-15}{11} \quad y = \frac{1}{11}x - \frac{15}{11}$

Write an equation for a horizontal line through $(8,-6)$ $\quad y = -6$

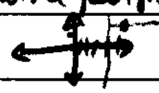
Write an equation for a vertical line through $(8,-6)$ $\quad x = 8$

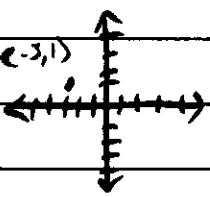
Algebra 3.3

EX Find the equation for a line parallel (//) to the y-axis thru (3,5)

$x = 3$ 

EX Find the line perpendicular ($\perp$) to $x=4$ thru (5,2)

$y = 2$ 



EX Find a general/standard form of an equation that's parallel to $y = \frac{2}{3}x + 7$ thru $(-3,1)$ (parallel lines have same slope) $m = \frac{2}{3}$

$y = \frac{2}{3}x + b \quad 1 = \frac{2}{3}(-3) + b \quad 1 = -2 + b \quad b = -3 \quad y = \frac{2}{3}x + 3 \quad -\frac{2}{3}x + y = 3$

$2x - 3y = -9$ standard/general form

EX Find the general form of the equation perpendicular to $y = 3x - 1$ thru $(5,-1)$ (perpendicular lines have opposite/reciprocal slopes)

$y = 3x - 1$ // slope $= 3$ $\perp$ slope $= -\frac{1}{3}$

$y = -\frac{1}{3}x + b \quad -1 = -\frac{1}{3}(5) + b \quad -1 = -\frac{5}{3} + b \quad b = \frac{2}{3}$

$y = -\frac{1}{3}x + \frac{2}{3} \quad \frac{1}{3}x + y = \frac{2}{3} \quad x + 3y = 2$

EX Find the equation for the perpendicular bisector of the segment AB.

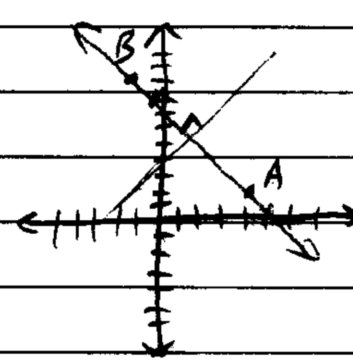
$A(4,2) \quad B(-2,10)$

$M = \left(\frac{4-2}{2}, \frac{2+10}{2}\right) = (1,6)$

$m = \frac{10-2}{-2-4} = \frac{8}{-6} = -\frac{4}{3} \qquad \perp m = \frac{3}{4}$

$y = \frac{3}{4}x + b \quad 6 = \frac{3}{4}(1) + b \quad b = \frac{21}{4}$

$y = \frac{3}{4}x + \frac{21}{4}$



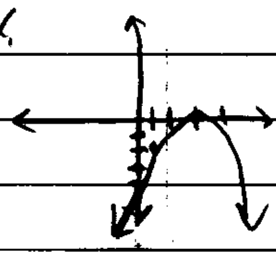
Algebra 3.4 Definition of Functions

Ex. $f(x)=3x-2$ same as $y=3x-2$ so $(x,f(x))$ same as (x,y)

Ex Find $f(-2)$ for $f(x)=x^2-3x-5$

$f(-2)=(-2)^2-3(-2)-5$ $\quad 4+6-5=5$ $\quad f(-2)=5$ or $(-2,5)$

Ex.



This is a function because it passes the verticle line test, ie verticle line passes thru it once & only once

Find $f(0)$ $\quad f(0)=-4$

Find $f(1)$ $\quad f(1)=-2$

Find $f(3)$ $\quad f(3)=0$

Ex Find $g(\frac{a}{2})$ for $g(x)=x^3-3x$

$g(\frac{a}{2})=(\frac{a}{2})^2-3(\frac{a}{2})$ $\quad \frac{a^3}{8}-\frac{3a}{2}$ $\quad g(\frac{a}{2})=\frac{a^3-12a}{8}$

<u>Domain</u> - is all possible x values that can be used

<u>Range</u> - is all possible y values that can be used

Ex Find the domain of $f(x)=\frac{2}{x}$ $\quad x\neq 0$ $\quad (-\infty,0)\cup(0,\infty)$

Ex Find the domain of $f(x)=\frac{11}{3x^2-2x-5}$ $\quad x\neq -1, \frac{5}{3}$ $\quad (-\infty,-1)\cup(-1,\frac{5}{3})\cup(\frac{5}{3},\infty)$

Ex Find the domain of $f(x)=\frac{3x}{\sqrt{x-4}}$ $\quad x-4>0$ $\quad x>4$ $\quad (4,\infty)$

Ex $h(x)=3x\sqrt{x-4}$ $\quad x-4\leq 0$ $\quad x\geq 4$ $\quad [4,\infty)$

Ex $f(x)=\frac{-7x^2}{(x+3)\sqrt{2-x}}$ $\quad x\neq -3$ $\quad 2-x>0$ $\quad -x>-2$ $\quad x<2$ $\quad (-\infty,3)\cup(-3,2)$

EX Find the domain of $\frac{-5x^3}{\sqrt{x^2-9}}$ $\quad x^2-9>0$

$(x+3)(x-3)>0$

$x-3$	$-$	$-$	$+$
$x+3$	$-$	$+$	$+$
all	$+$	$-$	$+$
	-3	3	

$(-\infty,3)\cup(3,\infty)$

EX Find the domain of $f(x)\ \frac{\sqrt{x-2}}{x+5}$ $\quad x-2\geq 0 \quad x\geq 2$ $\quad x\neq -5$ $\qquad [2,\infty)$

EX Find the domain of x^2-3x+4 $\quad \mathbb{R}$

EX Find the domain of $\frac{1}{x^2-3x-4}$ $\quad \frac{1}{(x-4)(x+1)}$ $\quad x\neq 4,-1$ $\quad (-\infty,-1)\cup(-1,4)\cup(4,\infty)$

EX $f(x)=\frac{9x}{\sqrt{x^3-4x}}=\frac{9x}{\sqrt{x(x^2-2)}}=\frac{9x}{\sqrt{x(x-2)(x+2)}}$ $\quad x\neq 0,2,-2$

$(-2,0)\cup(2,\infty)$

x	$-$	$-$	$+$	$+$
$x-2$	$-$	$+$	$+$	$+$
$x+2$	$-$	$-$	$-$	$+$
all	$-$	$+$	$-$	$+$
	-2	0	2	

EX $g(x)=\sqrt{x^3-4x}$ $\qquad [-2,0]\cup[2,\infty)$

EX if $f(-2)=7$ and $f(4)=-2$, find the linear function.

$(-2,7)$ and $(4,-2)$ $\quad m=\frac{-2-7}{4--2}=\frac{-9}{6}=-\frac{3}{2}$

$y=mx+b$

$y=-\frac{3}{2}(x)+b$ $\qquad y=-\frac{3}{2}x+4$

$7=-\frac{3}{2}(-2)+b$ $\qquad f(x)=-\frac{3}{2}x+4 \leftarrow$ function notation

$7=3+b \quad b=4$

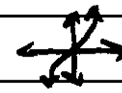
Remember:

For all $\sqrt{\ }$ we don't want negatives!

For $\sqrt{\ }$ in the denominator, we don't want 0 or negatives!

Algebra 3.5 Graphs of Functions

Even Function $f(-x) = f(x)$ $y = x^2$ 

Odd Function $f(-x) = -f(x)$ $y = x^3$ 

Determine whether the following are even, odd or neither

a) $f(x) = 5x^3 + 2x$
$f(-x) = 5(-x)^3 + 2(-x)$
$= -5x^3 - 2x$ odd

b.) $f(x) = |x| - 3$
$f(-x) = |-x| - 3$
$= |x| - 3$ even

c) $f(x) = 10$
$f(-x) = 10$
even

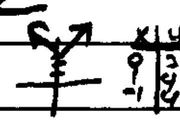
d) $f(x) = 3x^2 - 5x + 1$
$f(-x) = 3(-x)^2 - 5(-x) + 1$
$3x^2 + 5x + 1$ neither

e) $f(x) = x^4 - 7x^2$
$f(-x) = (-x)^4 - 7(-x)^2$
$= x^4 - 7x^2$ even

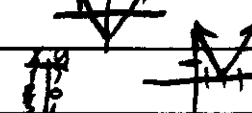
f) $f(x) = x^3 - 2x + 3$
$f(-x) = (-x)^3 - 2(-x) + 3$
$-x^3 + 2x + 3$ neither

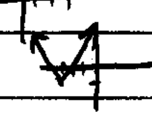
Absolute Value Function

General graph: $y = |x|$ 

graph: $y = |x| + 3$ 

graph: $y = |x| - 5$ shift down 5 

Graph: $y = |x - 2|$ shift right 2 

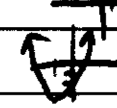
graph: $y = |x+4| - 1$ shift left 4, down 1 

Basic Parabola

General Graph: $y = x^2$ 

$y = x^2 + 6$ shift up 6 units 

$y = (x-3)^2$ shift right 3 units 

$y = (x+1)^2 - 4$ shift left 1, down 4 

Cubic

General Graph $y = x^3$  $y = -x^3$ 

Alg 3.5

$f(x) = -x^2 + 5$ 

EX If the point (2,7) is on the graph of f. Find the corresponding point on the graph of the function
$y = f(x-1)+5$ right 1, up 5 $(2,7) \Rightarrow (3,12)$

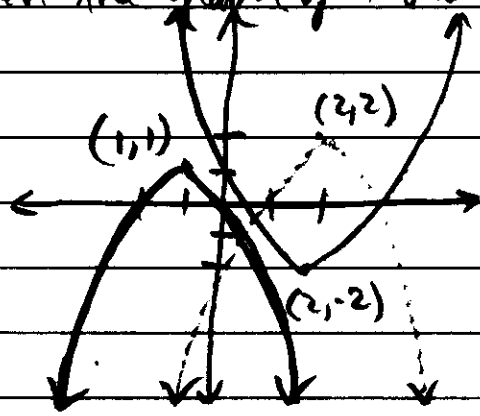
EX If the point (-2,3) is on the graph of f. Find the corresponding point on the graph of the function
$y = -f(x+4)-1$ left 4, down 1

EX Sketch the graph of $y = -f(x+3) - 1$
given the graph of f below

left 3

down 1

reflect & flip about the x-axis



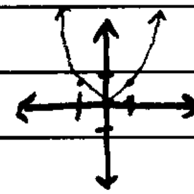
Algebra 3.6 Quadratics

$ax^2 + bx + c$ vertex minimum, vertex maximum $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

axis of symmetry $x = \frac{-b}{2a}$

Ex graph $f(x) = \frac{1}{2}x^2$

x	y
-1	1/2
0	0
1	1/2



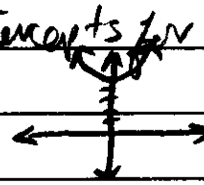
$g(x) = 2x^2$

x	y
-1	2
0	0
1	2

Ex Find the vertex & x intercepts for the following:

a) $f(x) = x^2 + 5$

$\left(\frac{0}{2a}, f(0)\right) = (0, 5)$



no x intercepts

$0 = x^2 + 5$
$x^2 = -5$
$x = \pm\sqrt{-5}$
$\cancel{x}$ no

b) $g(x) = -x^2 + 4x$

vertex $\left(\frac{-4}{2(-1)}, f(2)\right) = (2, 4)$

$g(x) = -x^2 + 4x$
$= -(2)^2 + 4(2)$
$= -4 + 8$
$= 4$

x intercepts
$0 = -x^2 + 4x$
$0 = -x(x - 4)$
$x = 0, 4$

Standard Equation $y = a(x-h)^2 + k$ vertex (h, k)

Ex Write $y = x^2 + 4x + 9$ into standard equation

$y = x^2 + 4x \underline{+ 4} + 9 \underline{- 4}$

$y = (x+2)^2 + 5$

vertex: $(-2, 5)$

Write $y = -3x^2 - 6x - 5$ into standard equation

$y = -3(x^2 + 2x \underline{+1}) - 5 \underline{+ 3}$

-3

$y = -3(x+1)^2 - 2$

vertex: $(-1, -2)$ down shaped parabola, skinny

Algebra 3.6

Write $y = 2x^2 - 10x + 7$ into the standard equation

$y = 2(x^2 - 5x + \underline{\tfrac{25}{4}}) + 7 \underline{-\tfrac{25}{2}}$

$y = 2(x - \tfrac{5}{2})^2 - \tfrac{11}{2}$

vertex $(\tfrac{5}{2}, -\tfrac{11}{2})$ up shaped, skinny

Write $y = -\tfrac{3}{4}x^2 + 15x - 16$ into the standard equation

$-\tfrac{3}{4} \cdot ? = 15$

$? = 15(-\tfrac{4}{3}) = -20$

$y = -\tfrac{3}{4}(x^2 - 20x \underline{+100}) - 16 \underline{+75}$

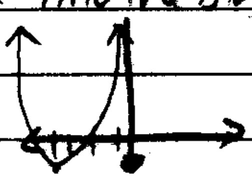
$y = -\tfrac{3}{4}(x - 10)^2 + 59$

vertex $(10, 59)$ open down, fat

Ex Find the standard equation for <u>any</u> parabolla w/vertex $(-6,3)$

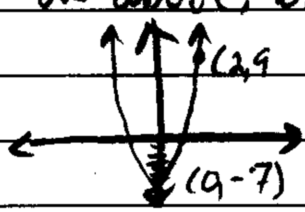
$y = \underline{a}(x+6)^2 + 3$

Ex Find the standard equation of the graph below: (assume $a = 1$)



$y = (x+4)^2 - 1$

Ex As above, except don't assume $a = 1$



$y = a(x+0)^2 - 7$

$y = ax^2 - 7 \Rightarrow y = 4x^2 - 7$

$9 = a(2)^2 - 7$

$9 = 4a - 7$

$16 = 4a$

$a = 4$

Algebra 3.6

Find the minimum value & the zeros of the function

$y = x^2 + 6x + 8$

$y = x^2 + 6x \underline{+9} + 8 \underline{-9}$

$y = (x+3)^2 - 1$

vertex $(-3,-1)$ (Min value)

$0 = x^2 + 6x + 8$

$0 = (x+4)(x+2)$

$x = -4, -2$ zeros are $-4, -2$

Ex An object is projected vertically upward with an initial velocity of 176 ft/sec.
It's distance in feet above ground after t seconds is given by the equation $s(t) = -16t^2 + 176t + 96$
Find the maximum height of the object

$s(t) = -16(t^2 - 11t + \frac{121}{4}) + 96 \underline{+484}$

$s(t) = -16(t - \frac{11}{2})^2 + 580$

vertex $(\frac{11}{2}, 580)$ (maximum value

max height is 580 feet

Algebra 3.7 Operations on Functions

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$(f/g)(x) = f(x)/g(x) \quad g(x) \neq 0$

EX. $f(x) = 4x^3 - x + 1 \qquad g(x) = 3x^2 + 2$

a) $(f+g)(x) \quad 4x^3 - x + 1 + 3x^2 + 2 = 4x^3 + 3x^2 - x + 3$

b) $(f+g)(2) \quad 4(2)^3 + 3(2)^2 - 2 + 3 = 32 + 12 - 2 + 3 = 45$

c) $(f-g)(x) \quad 4x^3 - x + 1 - 3x^2 - 2 = 4x^3 - 3x^2 - x - 1$

d) $(f-g)(-2) \quad 4(-2)^3 - 3(-2)^2 - (-2) - 1 = -32 - 12 + 2 - 1 = -43$

e) $(f*g)(x) \quad (4x^3 - x + 1)(3x^2 + 2) = 12x^5 + 5x^3 + 3x^2 - 2x + 2$

f) $(f/g)(x) \quad \dfrac{4x^3 - x + 1}{3x^2 + 2} \quad 3x^2 + 2 = 0 \quad 3x^2 = -2 \quad x^2 = -\frac{2}{3} \quad x = \pm\sqrt{-2/3}$ no imag so all $\mathbb{R}$

g) $(f/g)(1) \quad \dfrac{4(1)^3 - (1) + 1}{3(1)^2 + 2} = \dfrac{4 - 1 + 1}{5} = 4/5$

Let $f(x) = 3x - 5 \quad g(x) = x^2 - 4$

Find $(f/g)(x) = \dfrac{3x-5}{x^2-4} \quad x \neq 2, -2$

Find the domain of $(\frac{f}{g})(x)$, put in interval notation $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Find the domain of $(\frac{f}{g})(x) \quad f(x) = \sqrt{x+3} \quad g(x) = \sqrt{x+3}$

$(\frac{f}{g})(x) = \dfrac{\sqrt{x+3}}{\sqrt{x+3}} \rightarrow x + 3 > 0 \quad x > -3 \quad (-3, \infty)$

Find the domain of $(\frac{f}{g})(x) \quad f(x) = \sqrt{x+5} \quad g(x) = \sqrt{3-x}$

$(\frac{f}{g})(x) = \dfrac{\sqrt{x+5}}{\sqrt{3-x}} \quad \rightarrow x + 5 \geq 0 \quad x \geq -5$

$\rightarrow 3 - x > 0 \quad -x > -3 \quad x < 3 \qquad [-5, 3)$

Alg 3.7

Find the domain of $(fg)(x)$ $\quad f(x) = \sqrt{x+5} \quad g(x) = \sqrt{3-x}$

$$(fg)(x) = (\sqrt{x+5})(\sqrt{3-x})$$

$\hookrightarrow 3-x \geq 0 \quad -x \geq -3 \quad x \leq 3$

$\hookrightarrow x+5 \geq 0 \quad x \geq -5$

$[-5, 3]$

Find the domain of $(fg)(x)$ $\quad f(x) = \sqrt{x+4} \quad g(x) = \frac{7}{x}$

$$(fg)(x) = (\sqrt{x+4})\left(\frac{7}{x}\right) = \frac{7\sqrt{x+4}}{x} \leftarrow x \geq -4 \quad x \neq 0$$

$[-4, 0) \cup (0, \infty)$

Composition of Functions

$(f \circ g)(x) = f(g(x))$ $\quad$ it's read f composed of g

Ex Let $f(x) = x^2 + 2x$

$g(x) = \sqrt{x}$

Find $(f \circ g)(x)$

$$f(g(x)) = (\sqrt{x})^2 + 2(\sqrt{x}) = x + 2\sqrt{x}$$

Domain $= [0, \infty)$

Find $(g \circ f)(x)$

$$g(f(x)) = \sqrt{x^2 + 2x} \qquad \sqrt{x(x+2)}$$

test 0, 2 $\qquad$ $-2 \quad 0$

Domain $=$ $(-\infty, -2] \cup [0, \infty)$

Ex. Find a composite function form for y if $y = \frac{\sqrt{x-2}}{2 - \sqrt{x-2}}$

So let $f(x) = \frac{x}{2-x}$

$g(x) = \sqrt{x-2}$

Algebra 3.9 Variations

Variation - describes relationships between 2 or more variables
K - constant of proportionality

Direct variation - $y = Kx$ ex $d = rt$ time incr, distance incr
Indirect variation - $y = \frac{K}{x}$ ex $P = \frac{K}{V}$ pressure ↑, volume ↓

Express a statement as a formula that involves u, v, and a constant of proportionality. Also determine the value of K.

1) u is directly proportional to v, and if $v = 9$, $u = 18$
$u = Kv$ $18 = K \cdot 9$ $K = 2$ $u = 2v$

2) u is indirectly proportional to v, and $v = 2$, $u = 7$
$u = \frac{K}{v}$ $7 = \frac{K}{2}$ $K = 14$ $u = \frac{14}{v}$

3) r varies directly with s, & indirectly with the square of t
$r = \frac{Kt}{t^2}$ $3 = \frac{K \cdot 16}{4^2}$ $3 = \frac{16K}{16}$ $K = 3$ $r = \frac{3t}{t^2}$

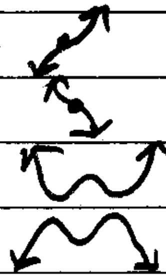
4.) r varies directly with the square root of s, and indirectly with the sum of s & t, find K, $r = 10$, $s = 4$, $t = 8$
$r = \frac{K\sqrt{s}}{s+t}$ $10 = \frac{K\sqrt{4}}{4+8}$ $10 = \frac{2K}{12}$ $K = 60$ $r = \frac{60\sqrt{s}}{s+t}$

5.) r varies directly with the opposite of t and indirectly with the difference of s & t, $r = -5$, $s = 2$, $t = -10$
$r = -\frac{Kt}{s-t}$ $-5 = \frac{-K(-10)}{2 - -10}$ $-5 = \frac{10K}{12}$ $K = -6$ $r = \frac{6K}{s-t}$

6.) r varies directly w/ the square of the sum of s & t, and indirectly w/ the cube root of t. $r = 1$, $s = 5$, $t = -8$
$r = \frac{K(s+t)^2}{\sqrt[3]{t}}$ $1 = \frac{K(5+-8)^2}{\sqrt[3]{-8}}$ $1 = \frac{9K}{-2}$ $K = -2/9$

Algebra 4.1 Polynomials of Degree greater Than 2

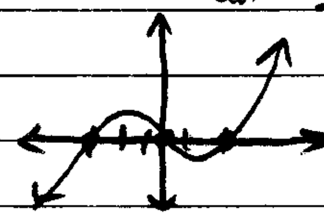
x^3 poly w/ a positive leading coefficient
x^3 poly w/ a negative leading coefficient
x^4 poly w/ a positive leading coefficient
x^4 poly w/ a negative leading coefficient



Ex $f(x) = 2x(x-2)(x+3)$ Test 0, 2, 3

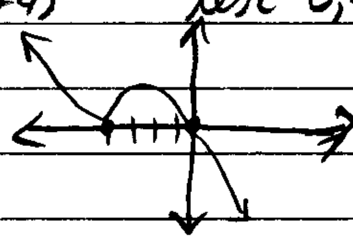
$2x$	−	−	+	+
$x-2$	−	−	−	+
$x+3$	−	+	+	+
all	−	+	−	+
		−3	0	2

When is $f(x) > 0$ $2x(x-2)(x+3) > 0$
when is $f(x) < 0$ $2x(x-2)(x+3) < 0$
$f(x) > 0$ $(-3,0) \cup (2,\infty)$
$f(x) < 0$ $(-\infty,-3) \cup (0,2)$



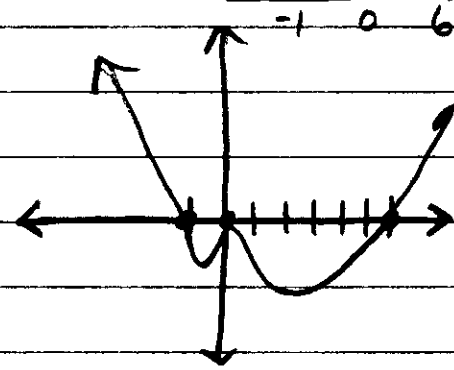
Ex $f(x) = -3x(x+4)^2$ test 0, −4

$-3x$	+	+	−
$(x+4)^2$	+	+	+
all	+	+	−
		−4	0



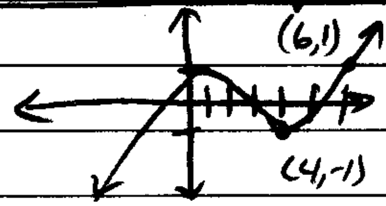
Ex $f(x) = x^4 - 5x^3 - 6x^2$
$x^2(x^2 - 5x - 6)$
$x^2(x-6)(x+1)$
zeroes 0, 6, −1

x^2	+	+	+	+
$x-6$	−	−	−	+
$x+1$	−	+	+	+
all	+	−	−	+
		−1	0	6



Alg 4.1

Intermediate Value Theorem



Somewhere between 4 and 6, y must equal zero

Ex Does $f(x) = x^5 + 4x^4 - 2x^3 + 3x - 7$ have a zero between 1 & 2?

Plug 1 and 2 in for x and see what happens

$f(1) = (1)^5 + 4(1)^4 - 2(1)^3 + 3(1) - 7$

$= 1 + 4 - 2 + 3 - 7 = -1$ $(1,-1)$

$f(2) = (2)^5 + 4(2)^4 - 2(2)^3 + 3(2) - 7$

$= 32 + 64 - 16 + 6 - 7 = 79$ $(2,79)$ yes

If $f(x) = x^3 - 5x^2 - 9x + 15K$ has 5 for a zero, find 2 other zeros

$0 = (5)^3 - 5(5)^2 - 9(5) + 15K$

$0 = 125 - 125 - 45 + 15K$

$K = 3$

$f(x) = x^3 - 5x^2 - 9x + 45$

$x^2(x-5) - 9(x-5)$

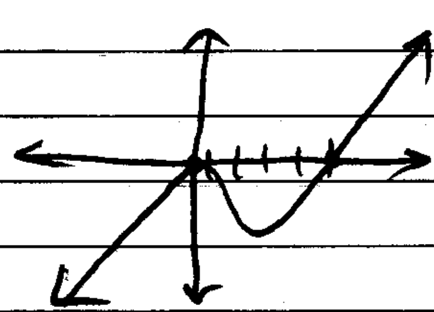
$(x^2-9)(x-5)$

$(x+3)(x-3)(x-5)$ the other two zeros are 3 and -3

EX $f(x) = x^3 - 5x^2$

$x^2(x-5)$

x^2	+	+	+
$x-5$	−	−	+
all	−	−	+
		0	5



Algebra 4.2 Properties of Division

Long Division Divide $3x^2-2x-7$ by $x+1$

quotient: $3x-5$
remainder: -2

$$
\begin{array}{r}
3x-5 \\
x+1 \overline{)\,3x^2-2x-7} \\
-(3x^2+3x) \\
\hline
-5x-7 \\
-(-5x-5) \\
\hline
-2
\end{array}
$$

Synthetic division Divide $3x^2-2x-7$ by $x+1$

$$
\begin{array}{r|rrr}
-1 & 3 & -2 & -7 \\
 & \downarrow & -3 & 5 \\
\hline
 & 3 & -5 & -2
\end{array}
$$

quotient: $3x-5$ rem: -2

Long Division Divide $5x^3-2x+4$ by x^2-3

can't use synthetic division with powers greater than 2

$$
\begin{array}{r}
5x \\
x^2+0x-3 \overline{)\,5x^3+0x^2-2x+4} \\
-(5x^3+0x^2-15x) \\
\hline
13x+4
\end{array}
$$

$\boxed{q: 5x \quad r: 3x+4}$

<u>Remainder Theorm</u> - If $f(x)$ is divided by $x-c$ then remainder is $f(c)$

EX. $f(x) = 3x^2-2x-7 \div x+1$ Use remainder thm to find rem
$c=-1 \quad f(-1)=3(-1)^2-2(-1)-7=3+2-7=\underline{-2}$

EX Use rem. thm to find the remainder when
$f(x)=2x^3-5x^2+4x+9 \div g(x)=x-4$
$c=4 \quad f(4)=2(4)^3-5(4)^2+4(4)+9=128-80+16+9=\underline{73}$

Alg 4.2

Factor Theorm if $x-c$ is a factor of $f(x)$ then $f(c)=0$

EX Is $x-2$ a factor of $f(x)=x^3-8$?

$c=2 \quad f(2)=2^3-8 \quad f(2)=0$ yes

EX Is $x+5$ a factor $f(x)=3x^2-7x+25$?

$c=-5 \quad f(-5)=3(-5)^2-7(-5)+25=75+35+25=125$ no

EX Find a polynomial f(x) of degree 3 w/ zeros 0, -1, 3 with a leading coefficient of 2

$f(x)=2x(x+1)(x-3)$

$=2x(x^2-2x-3)=\boxed{2x^3-4x^2-6x}$

EX Find any polynomial f(x) of degrees w/ zeros -4, with multiplicity of 2 and 7

$f(x)=a(x+4)^2(x-7)$

EX Use synthetic division to decide whether $x-3$ is a factor of x^4-2x^2+5

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -2 & 0 & 5 \\ & \downarrow & 3 & 9 & 21 & 63 \\ \hline & 1 & 3 & 7 & 21 & 68 \end{array}$$

$x-3$ is not a factor

EX Find all values of K such that $f(x)=Kx^3+x^2+K^2x+3K^2+11$ is divisible by $x+2$.

$c=-2 \quad 0=K(-2)^3+(-2)^2+K^2(-2)+3K^2+11$

$0=-8K+4-2K^2+3K^2+11$

$0=K^2-8K+15$

$0=(K-3)(K-5) \quad K=5,3$

Algebra 4.3 Zeros of Polynomials

Find the following polynomials w/ their given conditions

① zeros: 4, 1, -3 $\quad f(-1)=100$

$f(x) = a(x-4)(x-1)(x+3)$

$100 = a(-1-4)(-1-1)(-1+3)$

$100 = a(-5)(-2)(2) \quad 100 = 20a \quad a = 5$

$\underline{f(x) = 5(x-4)(x-1)(x+3)}$ ✓ stop here on tests

$f(x) = (5x-20)(x^2+2x-3)$

$f(x) = 5x^3+10x^2-15x-20x^2-40x+60$

$\underline{f(x) = 5x^3-10x^2-55x+60}$ ✓ stop here for iLrn

② zeros: 4, 2i, -2i $\quad f(-1)=-125$

$f(x) = a(x-4)(x-2i)(x+2i)$

$-125 = a(-1-4)(-1-2i)(-1+2i)$

$-125 = a(-5)(1-4i^2)$

$-125 = a(-5)(5)$

$-125 = -25a \quad a = 5$

$f(x) = 5(x-4)(x-2i)(x+2i)$ ✓ stop here for tests, multiply for iLrn

③ zeros: -3, -4 w/ multiplicity of 2 for both, leading coefficient = -1

$f(x) = \underline{-1}(x+3)^2(x+4)^2$ ✓ stop here for tests, multiply for iLrn

④ zeros: -1, 2 both multiplicity of 2, 0 multiplicity of 3, $f(2)=144$

$f(x) = a(x+1)^2(x-2)^2x^3$

$144 = a(2+1)^2(2-1)^2 2^3$

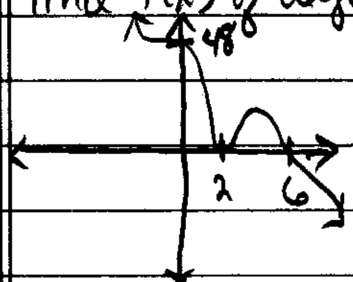
$144 = a(9)(1)(8)$

$144 = 72a \quad a = 2$

$f(x) = 2x^3(x+1)^2(x-1)^2$

Alg 4.3 cont.

Find $f(x)$ of degree 3, with it's graph below



$f(0)=48$

$$f(x)=a(x-2)^2(x-6)$$
$$48=a(0-2)^2(0-6)$$
$$48=a(4)(-6)$$
$$48=-24a \qquad a=-2$$
$$f(x)=-2(x-2)^2(x-6)$$

Find the zeros of $f(x)$ & state the multiplicity of each zero

$f(x)=-2x^3(x^2-8x+15)^3(x^2-9)^2$

$-2x^3(x-5)^3(x-3)^3(x+3)^2(x-3)^2$

zero	multiplicity
0	3
5	3
3	5
-3	2

Show that 3 is a zero of multiplicity of 2, and express $f(x)$ as a product of linear factors $f(x)=x^4-10x^3+33x^2-36x$

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 33 & -36 \\ & \downarrow & 3 & -21 & 36 \\ \hline 3 & 1 & -7 & 12 & 0 \\ & \downarrow & 3 & -12 & \\ \hline & 1 & -4 & 0 & \end{array}$$

$$\boxed{f(x)=(x-3)^2(x-4)}$$

Algebra 4.4 Complex & Rational Zeros of Polynomials

Ex Factor x^3-8 and find it's zeros

$(x^3-8) = (x-2)(x^2+2x+4)$ use quadratic formula

$x = \frac{-2 \pm \sqrt{4-4(1)(4)}}{2}$

$= \frac{-2 \pm \sqrt{4-16}}{2}$

$= \frac{-2 \pm \sqrt{-12}}{2}$

$= \frac{-2 \pm 2i\sqrt{3}}{2}$

$= -1 \pm i\sqrt{3}$

zeros are $2, -1 \pm i\sqrt{3}$

Ex Find a polynomial $f(x)$ of degree 4 that has all real coefficients, and has zeros $3+i$, $-2i$

also $3-i$, $2i$ are zeros

$f(x) = (x-(3+i))(x-(3-i))(x+2i)(x-2i)$

$f(x) = (x-3-i)(x-3+i)(x^2+4)$

$f(x) = x^2-3x+ix-3x+9-3i-ix+3i-i^2\,(x^2+4)$

$f(x) = x^2-6x+10\,(x^2+4)$ ✓ stop here when i's are gone

Short cut $(x-(a+bi))(x-(a-bi))$ eg $(x-(3+i))(x-(3-i))$

$x^2-2ax+a^2+b^2$ $x^2-2(3)x+9+1$

$x^2-6x+10$

If a polynomial $f(x)$ of degree 2 has real coefficients and $-4+3i$ is a zero, find $f(x)$

also $-4-3i$ is a zero

$f(x) = (x-(-4+3i))(x-(-4-3i))$

$f(x) = x^2-2(-4)x+16+9$

$f(x) = x^2+8x+25$

Alg 4.4 cont.

If polynomial $f(x)$ of degree 3 has real coefficient, and zeros $-5, 5+2i$ also $5-2i$ is a zero

$f(x)=(x+5)(x-(5+2i))(x-(5-2i))$

$=(x+5)(x^2-2(5)x+25+4)$

$=(x+5)(x^2-10x+29)$

<u>Rational Root Theorm</u> If $f(x)=\overset{d}{3}x^2+x\overset{c}{-10}$

Then all possible rational roots c/d.

Thus c is a factor of -10, and d is a factor of 3.

List all possible values for c/d

$c=\pm1, \pm2, \pm5, \pm10$

$d=\pm1, \pm3$

$c/d=\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

actual zeros: $(3x-5)(x+2)$ $x=5/3, -2$

EX. Find the zeros of $x^3-x^2-10x-8$

List all possible values of $c=\pm1, \pm2, \pm4, \pm8$

all possible values of $d=\pm1$

all possible values of $c/d=\pm1, \pm2, \pm4, \pm8$

Try 2
$$\begin{array}{r|rrrr} 2 & 1 & -1 & -10 & -8 \\ & \downarrow & 2 & 2 & -16 \\ \hline & 1 & 1 & -8 & \end{array}$$

Try -2
$$\begin{array}{r|rrrr} -2 & 1 & -1 & -10 & -8 \\ & \downarrow & -2 & 6 & 8 \\ \hline & 1 & -3 & -4 & 0 \checkmark \end{array}$$

$(x+2)(x^2-3x-4)$

$(x+2)(x-4)(x+1)$

$x=-2, 4, -1$

Alg 4.4 cont.

Ex Find all zeros of $x^4+2x^3-15x^2-14x+56$

$c = \pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

$d = \pm1$

$c/d = \pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

try

$$\begin{array}{r|rrrrr} 2 & 1 & 2 & -15 & -14 & 56 \\ & \downarrow & 2 & 8 & -14 & -56 \\ \hline & 1 & 4 & -7 & -28 & 0 \end{array}$$

$(x-2)(x^3+4x^2-7x-28)$

try

$$\begin{array}{r|rrrr} -4 & 1 & 4 & -7 & -28 \\ & \downarrow & -4 & 0 & 28 \\ \hline & 1 & 0 & -7 & \end{array}$$

$(x-2)(x+4)(x^2-7)$

$(x-2)(x+4)(x+\sqrt{7})(x-\sqrt{7})$

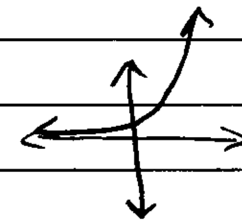
zeros are $2, -4, \sqrt{7}, -\sqrt{7}$

Algebra 5.1 Exponential Functions

Bases greater than 1 - exponential growth

$f(x) = a^x \quad a > 1$

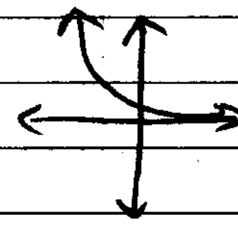
ex. - population, bacterial growth, compound interest



Bases less than 1 - exponential decay

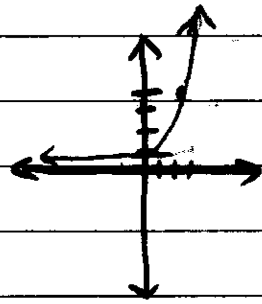
$f(x) = a^x \quad a < 1$

ex - radio active decay



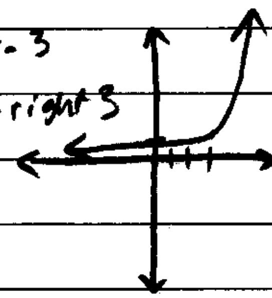
$f(x) = 2^x$

x	y
-3	1/8
-2	1/4
0	1
2	4
3	8



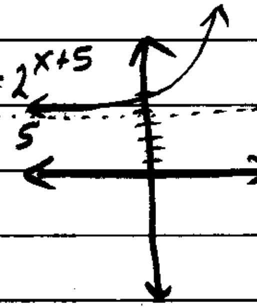
$g(x) = 2^{x-3}$

shift right 3



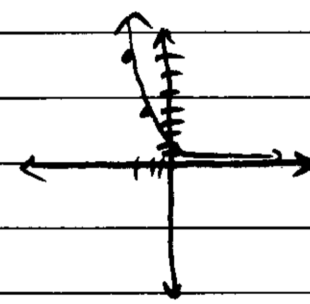
$h(x) = 2^{x+5}$

shift up 5



Graph $f(x) = \left(\frac{1}{2}\right)^x$

x	f(x)
-3	8
-2	4
0	1
2	1/4
3	1/8



Solve for x

① $2^{3x} = 2^{-7x-5}$

$3x = -7x - 5$

$10x = -5$

$x = -\frac{1}{2}$

② $3^{2x+3} = 3^{x^2}$

$2x + 3 = x^2$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1, 3$

③ $9^{x^2} = 3^{3x+2}$ ← Bases must be the same

$(3^2)^{x^2} = 3^{3x+2}$

$3^{2x^2} = 3^{3x+2}$

$2x^2 = 3x + 2$

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$x = -\frac{1}{2}, 2$

alg 5.1

④ $4^x \cdot \left(\frac{1}{2}\right)^{3-2x} = 8(2^x)^2$ — remember, the bases must be the same

$(2^2)^x \cdot (2^{-1})^{3-2x} = 2^3(2^{2x})$

$2^{2x} \cdot 2^{-3-2x} = 2^3 \cdot 2^{2x}$

$2^{4x-3} = 2^{3+2x}$

$4x-3 = 3+2x$

$2x = 6$

$x = 3$

Ex Find an exponential function of the form $f(x) = ba^x$ given y-int 8, and $P(3,1)$

y-int 8 = $(0,8)$

1st $f(x) = ba^x$

$8 = ba^0$

$8 = b \cdot 1$

$b = 8$

2nd $f(x) = ba^x$

$1 = 8a^3$

$a^3 = 1/8$

$a = \sqrt[3]{1/8}$

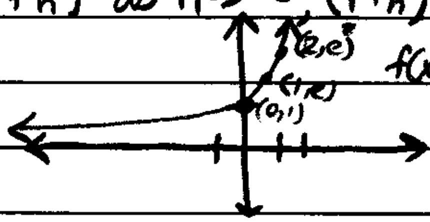
$a = 1/2$

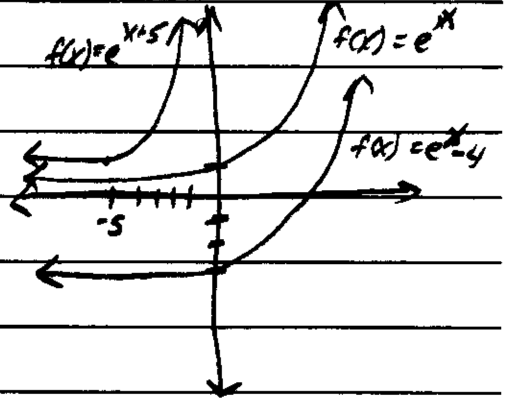
$f(x) = 8\left(\frac{1}{2}\right)^x$

Algebra 5.2 The Natural Exponential Function

Base $e \approx 2.71828$

$(1+\frac{1}{n})^n$ as $n \to \infty$, $(1+\frac{1}{n})^n$ approaches e





Continuously Compound Interest $A = Pe^{rt}$

ex you are investing \$20,000 over 6 years w/ 7% interest compounded continuously. Find the amount in your account at the end of 6 years.

$A = 20{,}000e^{(.07)(6)}$

$A = 20{,}000e^{.42}$

$A = 20{,}000(1.52196)$

$A = \$30{,}439.^{23}$

Law of Growth or Decay $q(t) = q_0 \cdot e^{rt}$ q_0 - initial quantity

ex Population of the US in 1980 was apx. 227 million and has grown continuously at 0.7% per year. Predict the population in 2010 if this trend continues.

$q(t) = q_0 \cdot e^{rt}$

$q(30) = 227 \cdot e^{(.007)(30)}$

$= 227e^{.21}$

$= 227(1.233678)$

$= 280$ million

Algebra 5.2

Solve for x

① $e^{x^2} = e^{7x-12}$

$x^2 = 7x - 12$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$\boxed{x = 3, 4}$

② $e^{2x}\left(\frac{1}{e^2}\right)^x e^{-4x} = e^6$

$e^{2x} e^{-2x} e^{-4x} = e^6$

$2x - 2x - 4x = 6$

$-4x = 6$

$\boxed{x = -\frac{3}{2}}$

Find the zeros of $f(x) = x^3(4e^{4x}) + 3x^2 e^{4x}$

(factor what's in common for both terms)

$x^3(4e^{4x}) + 3x^2 e^{4x}$

$\underbrace{x^2}_{x=0}\ \underbrace{e^{4x}}_{\text{does not produce zero}}\ \underbrace{(4x+3)}_{x=-3/4}$

$\boxed{x = 0, -3/4}$

Find the zeros of $f(x) = 12x^2 e^{2x} - 6xe^{2x}$

$\underbrace{6x}_{x=0}\ \underbrace{e^{2x}}_{\text{does not produce a zero}}\ \underbrace{(2x-1)}_{x=1/2}$

$\boxed{x = 0, 1/2}$

Algebra 5.3 Log Functions

Def of $\log_a$: $\log_a x = y$ same as $a^y = x$

log form - $\log_2 8 = y$ exponential form - $2^y = 8$

Change the following to exponential form

1) $\log_5 125 = x$ $5^x = 125$
2) $\log_3 x = 2$ $3^2 = x$
3) $\log_a 16 = 2$ $a^2 = 16$

Change the following to log form

1) $x^3 = 64$ $\log_x 64 = 3$
2) $10^5 = 100{,}000$ $\log_{10} 100{,}000 = 5$ or $\log 100{,}000 = 5$
3) $e^y = 2$ $\log_e 2 = y$ or $\ln 2 = y$ (natural log)

Change the following to exponential form

1) $\log 100 = 2$ $\log_{10} 100 = 2$ $10^2 = 100$
2) $\ln x = 5$ $\log_e x = 5$ $e^5 = x$
3) $\ln 3 = x-1$ $\log_e 3 = x-1$ $e^{x-1} = 3$

$\boxed{\ln e = 1}$ always replace $\ln e$ with 1

$\boxed{\ln 1 = 0}$ always replace $\ln 1$ with 0, same as $e^0 = 1$

$\boxed{\log 10 = 1}$ always replace $\log 10$ with 1

$\boxed{\log_a 1 = 0}$

Alg 5.3

Solve the following

① $\log_3 81$

$\log_3 81 = y$

$3^y = 81$

$y = 4$

② $\log 10^{-7}$

$\log_{10} 10^{-7}$

$10^y = 10^{-7}$

$y = -7$

③ $\log_x 64 = 3$

$x^3 = 64$

$x = 4$

④ $\log_x \frac{1}{64} = 3$

$x^3 = \frac{1}{64}$

$x = \frac{1}{4}$

⑤ $\log_3 \frac{1}{27}$

$\log_3 \frac{1}{27} = y$

$3^y = \frac{1}{27}$

$y = -3$

⑥ $\ln e^4$

$\ln e^4 = y$

$e^y = e^4$

$y = 4$

Solve for t

$3a^{t/2} = 10$

$a^{t/2} = 10/3$ (change to log form)

$\log_a \frac{10}{3} = \frac{t}{2}$

$2\log_a \frac{10}{3} = t$ (for iLrn $2 * \log(a, \frac{10}{2})$)

Solve for x.

$4 \cdot 3^{x-2} = 16$

$3^{x-2} = 4$ (change to log form)

$\log_3 4 = x - 2$

$x = 2 + \log_3 4$

Alg 5.3

Solve for x

$\log_3(x-7)=2$

$3^2=x-7$

$9+7=x$

$x=16$

Solve for x

$\log_7 x=\log_7(6-x)$

$x=6-x$

$2x=6$

$x=3$

Solve for x

$\log x^2=-8$

$10^{-8}=x^2$

$(\frac{1}{10})^8=x^2$

$\pm(\frac{1}{10})^4=x$ (for ihrn .0001, -.0001)

Find $\log_4 8$

$\log_4 8=y$

$4^y=8$

$(2^2)^y=2^3$

$2^{2y}=2^3$

$2y=3$

$y=3/2$

Algebra 5.4 Properties of Logs

$$\boxed{\begin{aligned}&\log_a(xy) = \log_a x + \log_a y\\ &\log_a\left(\tfrac{x}{y}\right) = \log_a x - \log_a y\\ &\log_a(x^c) = c\cdot\log_a x\end{aligned}}$$

Express in terms of x & y separately

① $\log_3(9x) = \log_3 9 + \log_3 x = 2 + \log_3 x$

② $\log_2\left(\frac{16}{y}\right) = \log_2 16 - \log_2 y = 4 - \log_2 y$

③ $\log(x^3) = 3\cdot\log x$

④ $\log_5(x^2y^3) = \log_5 x^2 + \log_5 y^3 = 2\log_5 x + 3\log_5 y$

⑤ $\ln\left(\frac{\sqrt{x}}{\sqrt[3]{y}}\right) = \ln\sqrt{x} - \ln\sqrt[3]{y} = \ln x^{1/2} - \ln y^{1/3} = \frac{1}{2}\ln x - \frac{1}{3}\ln y$

⑥ $\ln\sqrt[3]{\frac{x^5}{y^7z^4}} = \ln\frac{x^{5/3}}{y^{7/3}z^{4/3}} = \ln x^{5/3} - \ln y^{7/3} - \ln^{4/3} = \frac{5}{3}\ln x - \frac{7}{3}\ln y - \frac{4}{3}\ln z$

Common Mistakes

$\log(x+y) \neq \log x + \log y$

$\log(x-y) \neq \log x - \log y$

$\log x + \log y = \log z \qquad x+y \neq z$

Write the following as one logarithm

① $\ln 10x^2y^3 - \ln xy^5 = \ln\left(\frac{10x^2y^3}{xy^5}\right) = \ln\left(\frac{10x}{y^2}\right)$

② $\ln x^2 - \ln\left(\frac{1}{y}\right)^4 - \ln(xy)^3$

$\ln\left(\frac{x^2}{\left(\frac{1}{y}\right)^4(xy)^3}\right) = \ln\frac{x^2}{\frac{1}{y^4}x^3y^3} = \ln\left(\frac{1}{\frac{1}{y}x}\right) = \ln\left(\frac{y}{x}\right)$

Alg 5.4

(13) $9\log_2 x - 5\log_2\left(\frac{1}{y}\right) - 2\log_2(xy)$

$\log_2 x^9 - \log_2\left(\frac{1}{y}\right)^5 - \log_2(xy)^2$

$\frac{x^9}{\log_2\left(\frac{1}{y}\right)^5 (xy)^2} = \log_2 \frac{x^7}{\frac{1}{y^5}y^2} = \log_2 \frac{x^7}{\frac{1}{y^3}} = \boxed{\log_2 x^7 y^3}$

Solve the equation $\log(x+2) - \log x = 2\log 4$

$\log\left(\frac{x+2}{x}\right) = \log 4^2$

$\log\left(\frac{x+2}{x}\right) = \log 16$

$\frac{x+2}{x} = 16$

$x+2 = 16x$

$2 = 15x$

$x = 2/15$

Solve for z

$2\log_3 z = 3\log_3 5$

$\log_3 z^2 = \log_3 5^3$

$\log_3 z^2 = \log_3 125$

$z^2 = 125$

$z = \pm\sqrt{125} = \pm 5\sqrt{5}$ (one of these is wrong, toss out $-5\sqrt{5}$)

$= 5\sqrt{5}$

Alg 5.4

Solve for x $\log_2 x + \log_2(x+2) = 3$

$\log_2(x(x+2)) = 3$

$\log_2(x^2+2x) = 3$

$2^3 = x^2+2x$

$8 = x^2+2x$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4,2$ (can't be -4)

$\boxed{x=2}$

Solve for y

$\log_5(2x-5) = \log_5(10) - \log_5(2)$

$\log_5(2x-5) = \log_5(\frac{10}{2})$

$\log_5(2x-5) = \log_5(5)$

$2x-5=5$

$2x=10$

$\boxed{x=5}$

Write the following in terms of base 2

1. 1 $\log_2 \underline{2} = 1$
2. 3 $\log_2 \underline{8} = 3$
3. 5 $\log_2 \underline{32} = 5$
4. 0 $\log_2 \underline{1} = 0$
5. 4 $\log_2 \underline{\frac{1}{16}} = -4$ $(2^{-4} = \frac{1}{16})$

Solve for x $\log_2 x + 3 = \log_2(4x-1)$

$\log_2 x + \log_2 8 = \log_2(4x-1)$

$\log_2(8x) = \log_2(4x-1)$

$8x = 4x-1$

$4x=-1$

$\cancel{x} = -\frac{1}{4}$ no solution

Algebra 5.5 Solving Exp & Log Functions

Solve for x

$5^{-x} = 125$

$5^{-x} = 5^3$ ← same base

$-x = 3$

$\boxed{x = -3}$

Solve for x

$3^x = 11$ ← we can't have the same base for this problem, we must take natural log of both sides

$\ln 3^x = \ln 11$

$x \ln 3 = \ln 11$

$\boxed{x = \frac{\ln 11}{\ln 3}}$

Solve for x $\quad 2^{x-5} = 7$

$\ln 2^{x-5} = \ln 7$

$(x-5)\ln 2 = \ln 7$

$x - 5 = \frac{\ln 7}{\ln 2}$

$\boxed{x = \frac{\ln 7}{\ln 2} + 5}$

Solve for x $\quad 3^{2x} + 6(3^x) = 27$

use substitution - let $y = 3x$, and $y^2 = (3^x)^2 = 3^{2x}$

$3^{2x} + 6(3^x) = 27$

$y^2 + 6y = 27$

$y^2 + 6y - 27 = 0$

$(y+9)(y-3) = 0$

$y = -9, 3$ - we want to know what x is, not y, so plug y values in and solve.

$y = 3^x \qquad y = 3^x$

$-9 = 3^x \qquad 3 = 3^x$

no solution $\qquad \boxed{x = 1}$

Alg 5.5

Solve for x $\quad 4^x + 256 \cdot 4^{-x} = 68$

$4^x + 256\left(\frac{1}{4^x}\right) = 68$

$4^x\left(4^x + 256\left(\frac{1}{4^x}\right)\right) = 68\,(4^x)$ multiply both sides by 4x to get rid of the fraction

$4^{2x} + 256 = 68(4^x)$

$4^{2x} - 68(4^x) + 256 = 0$

use substitution - let $y = 4^x$, $y^2 = 4^{2x}$

$y^2 - 68y + 256 = 0$

$(y-64)(y-4) = 0$

$y = 64, 4$ - we want to know what x is, so plug y value in, and solve

$y = 4^x \qquad y = 4^x$

$64 = 4^x \qquad 4 = 4^x$

$x = 3 \qquad x = 1$

$\boxed{x = 1, 3}$

Solve for x $\quad \log x^3 = (\log x)^2$ — this is different from $\log x^2$

$3\log x = (\log x)^2$

$0 = (\log x)^2 - 3\log x$

$0 = \log x(\log x - 3)$ - (factored log x)

$\log x = 0 \qquad \log x - 3 = 0$

$10^0 = x \qquad \log x = 3$

$1 = x \qquad 10^3 = x$

$x = 1000 \qquad \boxed{x = 1, 1000}$

Solve for x $\quad (\log x)^4 = \log x^8$

$(\log x)^4 - \log x^8 = 0$

$(\log x)^4 - 8\log x = 0$

$\log x((\log x)^3 - 8) = 0$

$\log x = 0 \qquad (\log x)^3 - 8 = 0$

$10^0 = x \qquad (\log x)^3 = 8$

$1 = x \qquad \log x = 2$

$\boxed{x = 1} \qquad 10^2 = x$

$\boxed{x = 100}$

Alg 5.5

Solve for x & approx. to 2 decimal places

$\log(x^2+4)-\log(x+2)=2+\log(x-2)$

$\log(x^2+4)-\log(x+2)-\log(x-2)=2$

$\log\left(\frac{x^2+4}{(x+2)(x-2)}\right)=2$

$\log\left(\frac{x^2+4}{x^2-4}\right)=2$

$10^2=\frac{x^2+4}{x^2-4}$

$100=\frac{x^2+4}{x^2-4}$

$100(x^2-4)=x^2+4$

$100x^2-400=x^2+4$

$99x^2=404$

$x^2=\frac{404}{99}$

$x=\pm\sqrt{\frac{404}{99}}$

$x=\pm 2.02$ (toss out the negative value) $\boxed{x=2.02}$

Solve for x $\quad 2^{5x+3}=3^{2x+1}$

$\ln 2^{5x+3}=\ln 3^{2x+1}$

$(5x+3)\ln 2=(2x+1)\ln 3$

$5x(\ln 2)+3(\ln 2)=2x(\ln 3)+1(\ln 3)$

$5x(\ln 2)-2x(\ln 3)=\ln 3-3(\ln 2)$ - put x's on same side

$x(5(\ln 2)-2(\ln 3))=\ln 3-3(\ln 2)$ - factor the x

$x=\frac{\ln 3-3(\ln 2)}{5(\ln 2)-2(\ln 3)}$

$x=\frac{\ln 3-\ln 2^3}{\ln 2^5-\ln 3^2}$

$x=\frac{\ln 3-\ln 8}{\ln 32-\ln 9}$

$\boxed{x=\frac{\ln(\frac{3}{8})}{\ln(\frac{32}{9})}}$

Alg 5.5

Change of Base Formula ★★★ not on test

$$\log_a b = \frac{\log b}{\log a} \text{ or } \frac{\ln b}{\ln a}$$

appx $\log_2 20 = \frac{\log 20}{\log 2} = 4.32$

appx $\dfrac{\log_7 64}{\log_7 4} = \dfrac{\frac{\log 64}{\log 7}}{\frac{\log 4}{\log 7}} = \frac{\log 64}{\cancel{\log 7}} \cdot \frac{\cancel{\log 7}}{\log 4} = \frac{\log 64}{\log 4}$

$\approx \boxed{3}$

Guidelines for Solving Exponential or Logarithmic Equations

If you have a log equal to a number or a variable, change it to exponential form.

Ex: $\log_2 \frac{1}{16} = x$

$2^x = \frac{1}{16}$

$2^x = 2^{-4}$

$x = -4$

Ex: $\log_3 y = 4$

$3^4 = y$

$81 = y$

Ex: $\log_x 64 = 3$

$x^3 = 64$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

If you have a variable in your exponent, change it to logarithmic form.

Ex: $5^{2x-9} = 125$

$5^{2x-9} = 5^3$

$2x-9 = 3$

$2x = 12$

$x = 6$

Ex: $2^{3x-1} = \frac{1}{2}$

$2^{3x-1} = 2^{-1}$

$3x-1 = -1$

$3x = 0$

$x = 0$

If you have the same base on both sides of the equation, then simplify and set the exponents equal to each other.

Ex: $2^{x-3} = 2^{5x+13}$

$x-3 = 5x+13$

$-4x = 16$

$x = -4$

Ex: $(3^2)^x \cdot 3^{-x+1} = (3^{-1})^{3x-4} \cdot 3^2$

$2x - x + 1 = -3x + 4 + 2$

$4x = 5$

$x = 5/4$

If you don't have the same base on both sides of the equation, then try to get everything into the same base. Then, simplify and set the exponents equal to each other.

Ex: $(27)^x \cdot \left(\frac{1}{3}\right) = (9)^{x-4} \cdot 3^{-2}$

$3^{3x} \cdot 3^{-1} = 3^{2x-8} \cdot 3^{-2}$

$3x - 1 = 2x - 8 - 2$

$x = -9$

Ex: $4^{-2} \cdot (16^{2x})^3 = 8^{3x-2} \cdot \left(\frac{1}{32}\right)^{-x}$

$2^{-4} \cdot 2^{24x} = 2^{9x-6} \cdot 2^{5x}$

$-4 + 24x = 9x - 6 + 5x$

$10x = -2$

$x = -1/5$

If you don't have the same base on both sides of the equation, and you can't possibly get everything into the same base, then take the log or ln of both sides.

Ex: $7^{3x} = 11$

$\ln 7^{3x} = \ln 11$

$3x \ln 7 = \ln 11$

$3x = \frac{\ln 11}{\ln 7}$

$x = \frac{\ln 11}{3 \ln 7}$

Ex: $3^{x-2} = 14$

$\ln 3^{x-2} = \ln 14$

$(x-2) \ln 3 = \ln 14$

$x - 2 = \frac{\ln 14}{\ln 3}$

$x = \frac{\ln 14}{\ln 3} + 2$

If you have a radical (root) mixed in your equation, most of the time it helps to change it to a rational (fraction) exponent, and vice versa.

Ex: $\log_5 \sqrt[3]{5} = x$

$\log_5 5^{1/3} = x$

$5^x = 5^{1/3}$

$x = 1/3$

Ex: $\log_{16} x = \frac{3}{4}$

$16^{3/4} = x$

$(\sqrt[4]{16})^3 = x$

$2^3 = x$

$8 = x$

Ex: $\ln \sqrt[4]{e^3} = x$

$\ln e^{3/4} = x$

$\log_e e^{3/4} = x$

$e^x = e^{3/4}$

$x = 3/4$

If you have two logs of the same base set equal to one another, set the stuff in parenthesis (or that comes after the base) equal to each other.

Ex: $\log_2(x-3) = \log_2(9-5x)$

$x - 3 = 9 - 5x$

$6x = 12$

$\cancel{x=2}$

$\boxed{\text{no solution}}$

Ex: $\ln\sqrt{x-3} = \ln 7$

$\sqrt{x-3} = 7$

$x - 3 = 49$

$x = 52$

Ex: $\log x^2 = \log(6-x)$

$x^2 = 6 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = 2, -3$

If you have several logs dancing around on both sides of the equation, try to get them all on one side and use your rules for logs.

Ex: $\log_4(x-2) = 1 - \log_4(x+2) + \log_4 3$

$\log_4(x-2) + \log_4(x+2) - \log_4 3 = 1$

$\log_4 \frac{(x-2)(x+2)}{3} = 1$

$\log_4 \left(\frac{x^2-4}{3}\right) = 1$

$4^1 = \frac{x^2-4}{3}$

$12 = x^2 - 4$

$x^2 = 16$

$x = \pm 4$ $\quad x = 4$

Ex: $2\log_8 x + 4\log_8 2 = \log_8 x - 2$

$\log_8 x^2 + \log_8 2^4 - \log_8 x = -2$

$\log_8(16x^2) - \log_8 x = -2$

$\log_8\left(\frac{16x^2}{x}\right) = -2$

$\log_8 16x = -2$

$8^{-2} = 16x$

$\frac{1}{64} = 16x$

$\frac{1}{1024} = x$

If you have an equation that is quadratic in form, use substitution.

Ex: $5^x + 125\cdot(5^{-x}) = 30$

$5^x + 125(\frac{1}{5^x}) = 30$

$5^{2x} + 125 = 30\cdot 5^x$

$5^{2x} - 30\cdot 5^x + 125 = 0$

let $y = 5^x$

$y^2 - 30y + 125 = 0$

$(y-25)(y-5) = 0$

$y = 5, 25$

$y = 5^x$

$5 = 5^x \quad 25 = 5^x$

$x = 1 \quad x = 2$

$x = 1, 2$

Ex: $e^{2x} + 2e^x - 15 = 0$

let $e^x = y$

$y^2 + 2y - 15 = 0$

$(y+5)(y-3) = 0$

$y = -5 \quad y = 3$

$e^x = -5$

$\log_e -5 = x$

$\cancel{e^x = -5}$

$e^x = 3$

$\log_e 3 = x$

$x = \ln 3$

And sometimes you just need to use some good ole factoring.

Ex: $x^2e^{3x} = 7xe^{3x}$

$x^2e^{3x} - 7xe^{3x} = 0$

$xe^{3x}(x-7) = 0$

$xe^{3x} = 0 \quad x - 7 = 0$

$x = 0 \quad x = 7$

Ex: $(\log x)^4 = 27\log x$

$(\log x)^4 - 27\log x = 0$

$\log x\,((\log x)^3 - 27) = 0$

$\log x = 0$

$10^0 = x$

$x = 1$

$(\log x)^3 - 27 = 0$

$(\log x)^3 = 27$

$\log x = 3$

$10^3 = x \quad x = 1000$

$x = 1, 1000$

Algebra 9.1 Systems of Equations

Solve the following systems of equations using substitution

$$\begin{cases} y = x^2+1 \\ x+y=3 \end{cases} \Rightarrow x+x^2+1=3 \Rightarrow x^2+x-2=0 \Rightarrow (x+2)(x-1)=0$$

$$x=-2,1$$

$$y=x^2+1 \Rightarrow y=(-2)^2+1 \qquad y=(1)^2+1$$

$$y=5 \qquad y=2$$

write your final answer as coordinates

$\boxed{(-2,5),\ (1,2)}$ ← note: these points are where the lines intersect

Solve the following systems of equations using substitution

$$\begin{cases} x-y^3=1 \\ 2x=9y^2+2 \end{cases}$$

$x-y^3=1 \Rightarrow$ rewrite to get x by itself $\Rightarrow x=y^3+1$

$2x=9y^2+2 \Rightarrow 2(y^3+1)=9y^2+2 \Rightarrow 2y^3+2=9y^2+2$

$$2y^3-9y^2=0$$

$$y^2(2y-9)=0$$

$$y^2=0 \qquad 2y-9=0$$

$$y=0 \qquad y=9/2$$

$$x=y^3+1$$

$$x=(0)^3+1 \qquad x=\left(\tfrac{9}{2}\right)^3+1$$

$$x=1 \qquad x=\tfrac{729}{8}+1=\tfrac{737}{8}$$

$$(1,0),\ \left(\tfrac{737}{8},\tfrac{9}{2}\right)$$

Solve the following systems of equations using substitution

$$\begin{cases} x^2+y^2=25 \\ 3x+4y=-25 \end{cases} \Rightarrow 4y=-3x-25 \Rightarrow y=\frac{-3x-25}{4}$$

$$x^2+\left(\frac{-3x-25}{4}\right)^2=25 \Rightarrow x^2+\frac{9x+150x+625}{16}=25 \Rightarrow 16x^2+9x^2+150x+625=400$$

$$25x^2+150x+225=0 \Rightarrow 25(x^2+6x+9)=0 \Rightarrow 25(x+3)(x+3)=0 \Rightarrow x=-3$$

$$y=\frac{-3x-25}{4} \quad y=\frac{-3(-3)-25}{4} \quad y=\frac{-16}{4} \quad y=4$$

$$\boxed{(-3,-4)}$$

Algebra 9.1 cont.

Solve the following systems of equations using substitution

$$\begin{cases} xy = 2 \Rightarrow x = \frac{2}{y} \\ 6x - y + 4 = 0 \end{cases}$$

$6\left(\frac{2}{y}\right) - y + 4 = 0$

$12 - y^2 + 4y = 0$

$-y^2 + 4y + 12 = 0$

$y^2 - 4y - 12 = 0$

$(y-6)(y+2) = 0$

$y = 6, -2$

$x = 2/y$

$x = 2/6 \qquad x = \frac{2}{-2}$

$x = 1/3 \qquad x = -1$

$\boxed{(1/3, 6), (-1, -2)}$

Solve the following system of equations using substitution

$$\begin{cases} x^2 + 3y^2 = 13 \\ x^2 - y^2 = 12 \Rightarrow x^2 = y^2 + 12 \end{cases}$$

$y^2 + 12 + 3y^2 = 13$

$4y^2 = 1$

$y^2 = 1/4$

$y = \pm 1/2$

$x^2 = y^2 + 12$

$x^2 = (1/2)^2 + 12 \qquad x^2 = (-1/2)^2 + 12$

$x^2 = 1/4 + 48/4 \qquad x^2 = 1/4 + 48/4$

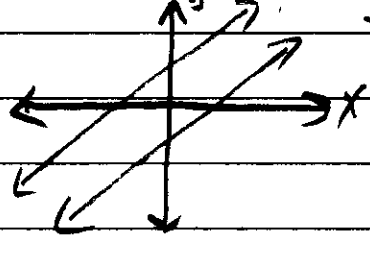
$x^2 = 49/4 \qquad x^2 = 49/4$

$x = \pm 7/2 \qquad x = \pm 7/2$

$\boxed{\left(\frac{7}{2}, \frac{1}{2}\right), \left(\frac{7}{2}, -\frac{1}{2}\right), \left(-\frac{7}{2}, \frac{1}{2}\right)\left(-\frac{7}{2}, -\frac{1}{2}\right)}$

Inconsistent system

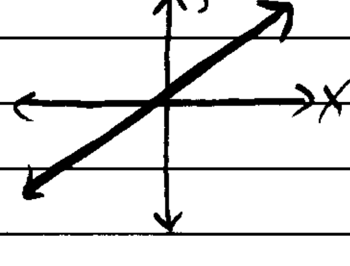
no solution



two parallel lines, never intersecting

Dependent system

infinite solutions



two overlapping lines, all points of each line intersect with the other.

Algebra 9.2 Systems of Linear Equations in Two Variables

Solving by elimination - multiply one or both equations by any number that will eliminate either x or y, when both equations are added together.

1) $\begin{cases} 5x - 8y = 11 \\ x + 8y = 7 \end{cases}$ Luckily, the $-8y$ and the $+8y$ eliminate each other, when adding the equations together.

$6x + 0 = 18$

$6x = 18$

$x = 3$ → $x + 8y = 7$ $3 + 8y = 7$ $8y = 4$ $y = \frac{4}{8} = \frac{1}{2}$

$\boxed{(3, \frac{1}{2})}$

2) $\begin{cases} 6x - 7y = 22 \\ 2x - 8y = -4 \end{cases}$ ← multiply by -3 to cancel x's

$6x - 7y = 22$

$-6x + 24y = 12$

$17y = 34$

$y = 2$

$2x - 8(-2) = -4$

$2x - 16 = -4$

$2x = 12$

$x = 6$

$\boxed{(6, 2)}$

3) $\begin{cases} 5x - 6y = 10 & \leftarrow \text{mult } 2 \\ 2x + 7y = 1 & \leftarrow \text{mult } -5 \end{cases}$

$10x - 12y = 20$

$-10x - 35y = -5$

$-47y = 15$

$y = -\frac{15}{47}$

$2x + 7(-\frac{15}{47}) = 1$

$2x - \frac{105}{47} = 1$

$2x = 1 + \frac{105}{47}$

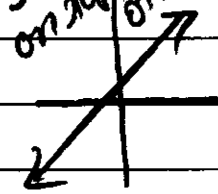
$2x = \frac{152}{47}$

$x = \frac{76}{47}$

$\boxed{(\frac{76}{47}, -\frac{15}{47})}$

Algebra 9.2 cont

same line on top of each other

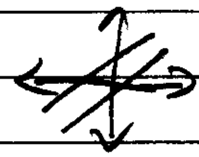


④ $\begin{cases} 6a - 7b = -11 \\ -18a + 21b = 33 \end{cases}$ ← mult by 3 $\quad$ $\begin{array}{r} 18a - 21b = -33 \\ \underline{-18a + 21b = 33} \\ 0 = 0 \text{ True} \end{array}$

for ilrn: dependent

$\boxed{\mathbb{R} \text{ or infinatily many solutions}}$

⑤ $\begin{cases} 9x - 24y = 15 \\ -3x + 8y = 7 \end{cases}$ (mult by 3) $\quad$ $\begin{array}{r} 9x - 24y = 15 \\ \underline{-9x + 24y = 21} \\ 0 = 36 \text{ False} \end{array}$



two parallel lines, no intersection

$\boxed{\emptyset \text{ or no solution}}$ for ilrn: inconsistant

Applied Problems

A factory makes desks and chairs. Each desk takes 12 hours to make, and each chair takes 5 hours to make. A desk cost \$175 each, and a chair costs \$95 each. If their is \$5085 and 329 labor hours available, how much of each can be made to use the full amount of hours & money.

let d = # of desks
let c = # of chairs

hours: $\begin{cases} 12d + 5c = 329 \\ 175d + 95c = 5085 \end{cases}$ (mult by -19) money:

$\begin{array}{r} -228d - 95c = -6251 \\ \underline{175d + 95c = 5085} \\ -53d \quad = -1166 \\ d = 22 \end{array}$

$12(22) + 5c = 329$
$264 + 5c = 329$
$5c = 65$
$c = 13$

$\boxed{22 \text{ desks } \& \text{ } 13 \text{ chairs}}$

Algebra 9.2 cont

Two bleach solutions have been made, one with 15% bleach, another with 25% bleach. How much should be combined from each to have 20 gallons of 22% solution?

let a = amt of 15% solution

let b = amt of 25% solution

$$\begin{cases} a+b=20 \\ .15a+.25b=.22(20) \end{cases}$$

mult line by -15 → $-15a-15b=-30$

multiply line by 100 to remove decimal → $15a+25b=440$

$10b=140$

$b=14$

$a+b=20$

$a+14=20$

$a=6$

14 gallons of 25% solution, 6 gallons of 15% solution

A witch plumping up children for eating is giving out chocolates & cream puffs. The chocolates has 34 grams of sugar & 17 grams of fat. The cream puffs has 27 grams of sugar & 16 grams of fat. How many of each should she feed the kids so that they consume 1668 grams of sugar and 849 grams of fat.

let c = # of chocolates

let p = # of puffs

$$\begin{cases} \text{sugar: } 34c+27p=1668 \\ \text{fat: } 17c+16p=894 \end{cases}$$

$34c+27p=1668$

$-34c-32p=-1788$

$-5p=-120$

$p=24$

$17c+16p=849$

$17c+16(24)=849$

$17c+384=849$

$17c=510$

$c=30$

24 cream puffs, 30 chocolates

Algebra 9.5 Systems of Linear Equations w/ More than 2 Variables
aka Solving Systems Using Matrices

Solve by elimination

$$\begin{cases} x-3y=1 \\ 2x+4y=7 \end{cases} \quad (-2)\ -2x+6y=-2$$

$$2x+4y=7$$

$$10y=5$$

$$y=\tfrac{1}{2}$$

$$x-3y=1$$
$$x-3(\tfrac{1}{2})=1$$
$$x-\tfrac{3}{2}=1$$
$$x=\tfrac{5}{2}$$

$$\left(\tfrac{5}{2},\tfrac{1}{2}\right)$$

The same problem using matrices

$$\left[\begin{array}{cc|c} 1 & -3 & 1 \\ 2 & 4 & 7 \end{array}\right] \xrightarrow{-2R1} \left[\begin{array}{cc|c} -2 & 6 & -2 \\ 2 & 4 & 7 \end{array}\right] \xrightarrow{R1+R2} \left[\begin{array}{cc|c} -2 & 6 & -2 \\ 0 & 10 & 5 \end{array}\right]$$

$$10y=5 \qquad y=\tfrac{1}{2}$$

$$x-3y=1$$
$$x-3(\tfrac{1}{2})=1$$
$$x=\tfrac{5}{2}$$

$$\left(\tfrac{5}{2},\tfrac{1}{2}\right)$$

EX

$$\begin{cases} x+3y-z=-3 \\ 3x-y+2z=1 \\ 2x-y+z=-1 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 3 & -1 & 2 & 1 \\ 2 & -1 & 1 & -1 \end{array}\right] \begin{array}{l} \\ -3R1+R2 \\ -2R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -10 & 5 & 10 \\ 0 & -7 & 3 & 5 \end{array}\right] \begin{array}{l} \\ 7R2 \\ -10R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -70 & 35 & 70 \\ 0 & 70 & -30 & -50 \end{array}\right] \xrightarrow{R2+R3} \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -70 & 35 & 70 \\ 0 & 0 & 5 & 20 \end{array}\right]$$

goal

$$5z=20$$
$$z=4$$

$$-70y+35z=70$$
$$-70y+35(4)=70$$
$$y=1$$

$$x+3y-z=-3$$
$$x+3(1)-(4)=-3$$
$$x=-2$$

$$(-2,1,4)$$

Operations you can use on matrices

1. Multiply or divide a row by a number
2. Interchange rows (We prefer having a 1 in top left corner)
3. Add two rows together to replace a row.

algebra 9.5 cont

EX

$$\begin{cases} 4x - y + 3z = 6 \\ -8x + 3y - 5z = -6 \\ 5x - 4y = -9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ -8 & 3 & -5 & -6 \\ 5 & -4 & 0 & -9 \end{array}\right] \; 2R1 + R2 \quad \left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ 0 & 1 & 1 & 6 \\ 5 & -4 & 0 & -9 \end{array}\right] \begin{array}{l} -5R1 \\ \\ 4R3 \end{array}$$

$$\left[\begin{array}{ccc|c} -20 & 5 & -15 & -30 \\ 0 & 1 & 1 & 6 \\ 20 & -16 & 0 & -36 \end{array}\right] \; R1 + R3 \quad \left[\begin{array}{ccc|c} -20 & 5 & -15 & -30 \\ 0 & 1 & 1 & 6 \\ 0 & -11 & -15 & -66 \end{array}\right] \; \frac{R1}{-5} \quad \left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & -11 & -15 & -66 \end{array}\right] \; 11R2 + R3$$

goal

$$\left[\begin{array}{ccc|c} 4 & -1 & 3 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & -4 & 0 \end{array}\right] \begin{array}{l} \to 4x - y + 3z = 6 \\ \to y + z = 6 \\ \to -4z = 0 \end{array}$$

$-4z = 0$
$z = 0$

$y + z = 6$
$y + (0) = 6$
$y = 6$

$4x - y + 3z = 6$
$4x - 6 + 3(0) = 6$
$4x = 12$
$x = 3$

$(3, 6, 0)$

EX

$$\begin{cases} x + 3y - 3z = -5 \\ 2x - y + z = -3 \\ -6x + 3y - 3z = 4 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -3 & -5 \\ 2 & -1 & 1 & -3 \\ -6 & 3 & -3 & 4 \end{array}\right] \begin{array}{l} \\ -2R1 + R2 \\ 6R1 + R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & -3 & -5 \\ 0 & -7 & 7 & 7 \\ 0 & 21 & -21 & -26 \end{array}\right] \; 3R2 + R3$$

goal

$$\left[\begin{array}{ccc|c} 1 & 3 & -3 & -5 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & -5 \end{array}\right] \to 0 = -5 \text{ False}$$

No solution

Algebra 9.5 cont.

EX

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 1 & -2 & -2 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \rightleftarrows \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ -2R1+R2 \\ -1R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right] -3R2+5R3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

goal

$\rightarrow x-2y-2z=0$

$\rightarrow 5y+5z=0$

$y+z=0$

$y=-z$

$x-2(-z)-2(z)=0$

$x+2z-2z=0$

$x=0$

$(0,-z,z)$

$0=0$

True (It means 'z' is a free variable, it can be anything it wants.)

EX

$$\begin{cases} x+y=3 \\ x-z=5 \\ x+z=2 \end{cases} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 1 & 0 & -1 & 5 \\ 0 & 1 & 1 & 2 \end{array}\right] \rightleftarrows \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & -1 & 5 \end{array}\right] -1R1+R3 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & 2 \end{array}\right] R2+R3$$

goal

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 4 \end{array}\right] \rightarrow 0=4$$

False

no solution

Algebra 9.8 Determinants of nxn Matrices

1x1 Matrix

$A = [2]$ $\quad$ $\det A = |A| = 2$

2x2 Matrix

$$A = \begin{array}{c} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

row
column

$\det A = |A| = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$

(cross multiply)

Ex $A = \begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix}$ $\quad |A| = (-5)(2) - (4)(-3) = -10 + 12 = 2$

Ex $A = \begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix}$ $\quad |A| = (6)(2) - (-4)(-3) = 12 - 12 = 0$

Ex $A = \begin{bmatrix} c & d \\ -d & c \end{bmatrix}$ $\quad |A = (c)(c) - (d)(-d) = c^2 + d^2$

3x3 Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad |A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Algebra 9.8 cont.

EX $A = \begin{bmatrix} -5 & 4 & 1 \\ 3 & -2 & 7 \\ 2 & 0 & 6 \end{bmatrix}$

$|A| = -5\begin{vmatrix} -2 & 7 \\ 0 & 6 \end{vmatrix} - 4\begin{vmatrix} 3 & 7 \\ 2 & 6 \end{vmatrix} + 1\begin{vmatrix} 3 & -2 \\ 2 & 0 \end{vmatrix}$

$= -5(-12-0) - 4(18-14) + 1(0+4)$

$= 60 - 16 + 4 = 48$

EX $A = \begin{bmatrix} 2 & -5 & 1 \\ -3 & 1 & 6 \\ 4 & -2 & 3 \end{bmatrix}$

$|A| = 2\begin{vmatrix} 1 & 6 \\ -2 & 3 \end{vmatrix} + 5\begin{vmatrix} -3 & 6 \\ 4 & 3 \end{vmatrix} + 1\begin{vmatrix} -3 & 1 \\ 4 & -2 \end{vmatrix}$

$= 2(3+12) + 5(-9-24) + 1(6-4)$

$= 2(15) + 5(-33) + 2 = -133$

4x4 Matrix

$A = \begin{bmatrix} 3 & -1 & 2 & 0 \\ 4 & 0 & -3 & 5 \\ 0 & 6 & 0 & 0 \\ 1 & 3 & -4 & 2 \end{bmatrix}$

$|A| = 3\begin{vmatrix} 0 & -3 & 5 \\ 6 & 0 & 0 \\ 3 & -4 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 4 & -3 & 5 \\ 0 & 0 & 0 \\ 1 & -4 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 & 5 \\ 0 & 6 & 0 \\ 1 & 3 & 2 \end{vmatrix} - 0$

$= 3\left[0\begin{vmatrix} 0 & 0 \\ 4 & 2 \end{vmatrix} + 3\begin{vmatrix} 6 & 0 \\ 3 & 2 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 3 & -4 \end{vmatrix}\right] + \left[4\begin{vmatrix} 0 & 0 \\ -4 & 2 \end{vmatrix} + 3\begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} + 5\begin{vmatrix} 0 & 0 \\ 1 & -4 \end{vmatrix}\right]$

$+ 2\left[4\begin{vmatrix} 6 & 0 \\ 3 & 2 \end{vmatrix} - 0 + 5\begin{vmatrix} 0 & 6 \\ 1 & 3 \end{vmatrix}\right]$

$= 3[0 + 3(12) + 5(-24)] + 1[0+0+0] + 2[4(12) + 5 - 6]$

$= 3[36 - 120] + 0 + 2(18)$

$= 3(-84) + 36$

$= -252 + 36$

$= -216$

Algebra 9.9 Properties of Determinants

1) If you interchange two rows in a matrix, you must change the sign of the determinant.

2) If you multiply/divide a row by a number, then you must multiply/divide the determinate by the number!

3.) If you multiply a row by a number and add it to another row, you don't change a thing!

Ex of Rule 1

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 5 & 0 \\ 3 & -2 & 5 \end{bmatrix} \updownarrow = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & 6 \\ 3 & -2 & 5 \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} 4 & 6 \\ -2 & 5 \end{vmatrix} - 5\begin{vmatrix} 2 & 6 \\ 3 & 5 \end{vmatrix} + 0$$
$$= 1(20+12) - 5(10-18)$$
$$= 32 - 5(-8) = 72$$

Change the sign! $\boxed{-72}$

Ex of Rule 2

$$A = \begin{bmatrix} 10 & 50 & -100 \\ 2 & -3 & 1 \\ 5 & 0 & 4 \end{bmatrix} \div(10) \begin{bmatrix} 1 & 5 & -10 \\ 2 & -3 & 1 \\ 5 & 0 & 4 \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} -3 & 1 \\ 0 & -4 \end{vmatrix} - 5\begin{vmatrix} 2 & 1 \\ 5 & -4 \end{vmatrix} - 10\begin{vmatrix} 2 & -3 \\ 5 & 0 \end{vmatrix}$$
$$= 1(12-0) - 5(-8-5) - 10(0+15)$$
$$= 12 - 5(-13) - 150$$
$$= 12 + 65 - 150 = -73$$

divide by 10! $\boxed{-\frac{73}{10}}$

Ex of Rule 3

$$A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & 7 & 1 \\ -6 & 0 & 4 \end{bmatrix} \begin{array}{l} \\ -2R1+R2 \\ 6R1+R3 \end{array} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 13 & -9 \\ 0 & -18 & 34 \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} 13 & -9 \\ -18 & 34 \end{vmatrix} + 3\begin{vmatrix} 0 & -9 \\ 0 & 34 \end{vmatrix} + 5\begin{vmatrix} 0 & 13 \\ 0 & -18 \end{vmatrix}$$
$$= 1(442-162) + 3(0) + 5(0)$$
$$= \boxed{280}$$

Algebra 9.9

Cramers Rule - take a system of equations, find D, D_x, D_y, D_z

$x = \frac{|D_x|}{|D|} \quad y = \frac{|D_y|}{|D|} \quad z = \frac{|D_z|}{|D|}$

EX Solve using Cramers Rule

$$\begin{cases} 4x + 5y = 13 \\ 3x + y = -4 \end{cases}$$

left hand side of equations

replace x column with right hand side of equation

replace y column with left hand side of equation

$D = \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix} \quad D_x = \begin{bmatrix} 13 & 5 \\ -4 & 1 \end{bmatrix} \quad D_y = \begin{bmatrix} 4 & 13 \\ 3 & -4 \end{bmatrix}$

$|D| = 4(1) - 5(3) = 4 - 15 = -11$

$|D_x| = 13(1) - 5(-4) = 13 + 20 = 33$

$|D_y| = 4(-4) - 13(3) = -16 - 39 = -55$

$x = \frac{|D_x|}{|D|} = \frac{33}{-11} = -3$

$y = \frac{|D_y|}{|D|} = \frac{-55}{-11} = 5 \quad \boxed{(-3, 5)}$

Special Cases

EX $x = \frac{3}{0}$, $y = \frac{3}{0}$, $z = \frac{3}{0}$ → denominators of zero mean inconsistent, no solution

EX $x = \frac{0}{0}$, $y = \frac{0}{0}$, $z = \frac{0}{0}$ → all values of zero mean infinately many sol. dependent $\mathbb{R}$

Algebra 9.9

Ex Solve Using Cramers Rule

$$\begin{cases} x+3y-z=-3 \\ 3x-y+2z=1 \\ 2x-y+z=-1 \end{cases}$$

$$D=\begin{bmatrix} 1 & 3 & -1 \\ 3 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \quad D_x=\begin{bmatrix} -3 & 3 & -1 \\ 1 & -1 & 2 \\ -1 & -1 & 1 \end{bmatrix}$$

$$D_y=\begin{bmatrix} 1 & -3 & -1 \\ 3 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \quad D_z=\begin{bmatrix} 1 & 3 & -3 \\ 3 & -1 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

$$D=\begin{bmatrix} 1 & 3 & -1 \\ 3 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\begin{matrix} \\ -3R1+R2 \\ -2R1+R3 \end{matrix}\begin{bmatrix} 1 & 3 & -1 \\ 0 & -10 & 5 \\ 0 & -7 & 3 \end{bmatrix}$$

$$|D| = 1\begin{vmatrix} -10 & 5 \\ -7 & 3 \end{vmatrix} - 3\begin{vmatrix} 0 & 5 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & -10 \\ 0 & -7 \end{vmatrix}$$
$$= 1(-30+35) = 5$$

flipped R1 & R2

$$D_x=\begin{bmatrix} 1 & -1 & 2 \\ -3 & 3 & -1 \\ -1 & -1 & 1 \end{bmatrix}\begin{matrix} \\ 3R1+R2 \\ R1+R3 \end{matrix}\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 5 \\ 0 & -2 & 3 \end{bmatrix}$$

$$|D_x| = 1\begin{vmatrix} 0 & 5 \\ -2 & 3 \end{vmatrix} = 1(0+10) = 10$$

change sign! $= -10$

$$D_y=\begin{bmatrix} 1 & -3 & -1 \\ 3 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\begin{matrix} \\ -3R1+R2 \\ -2R1+R3 \end{matrix}\begin{bmatrix} 1 & -3 & -1 \\ 0 & 10 & 5 \\ 0 & 5 & 3 \end{bmatrix}$$

$$|D_y| = 1\begin{vmatrix} 10 & 5 \\ 5 & 3 \end{vmatrix} = 1(30-25) = 5$$

$$D_z=\begin{bmatrix} 1 & 3 & -3 \\ 3 & -1 & 1 \\ 2 & -1 & -1 \end{bmatrix}\begin{matrix} \\ -3R1+R2 \\ -2R1+R2 \end{matrix}\begin{bmatrix} 1 & 3 & -3 \\ 0 & -10 & 10 \\ 0 & -7 & 5 \end{bmatrix}$$

$$|D_z| = 1\begin{vmatrix} -10 & 10 \\ -7 & 5 \end{vmatrix} = 1(-50+70) = 20$$

$$x = \frac{|D_x|}{|D|} = \frac{-10}{5} = -2$$
$$y = \frac{|D_y|}{|D|} = \frac{5}{5} = 1$$
$$z = \frac{|D_z|}{|D|} = \frac{20}{5} = 4$$

$(-2, 1, 4)$

Algebra 10.1 Infinite Sequences & Summation Notation

Find the first three terms of the following sequences & the 8th term

① $\{5-2n\}$ for $n=1,2,3\ldots$

$a_1 = 5-2(1) = 3$

$a_2 = 5-2(2) = 1$

$a_3 = 5-2(3) = -1$

$a_8 = 5-2(8) = -11$

② $\{1+(-1)^{n+1}\}$

$a_1 = 1+(-1)^{1+1} = 1+1 = 2$

$a_2 = 1+(-1)^{2+1} = 1+(-1)^3 = 1-1 = 0$

$a_3 = 1+(-1)^{3+1} = 1+(-1)^4 = 1+1 = 2$

$a_8 = 1+(-1)^{8+1} = 1+(-1)^9 = 1-1 = 0$

③ $\left\{\frac{2^n}{n^2+2}\right\}$

$a_1 = \frac{2^1}{1^2+2} = \frac{2}{3}$

$a_2 = \frac{2^2}{2^2+2} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$

$a_3 = \frac{2^3}{2^3+2} = \frac{8}{9+2} = \frac{8}{11}$

$a_8 = \frac{2^8}{8^2+2} = \frac{256}{64+2} = \frac{256}{66} = \frac{128}{33}$

④ $\{(-1)^n(2n+1)\}$

$a_1 = (-1)^1(2(1)+1) = -3$

$a_2 = (-1)^2(2(2)+1) = 5$

$a_3 = (-1)^3(2(3)+1) = -7$

$a_8 = (-1)^8(2(8)+1) = 17$

⑤ $\{7\}$

$a_1 = 7,\ a_2 = 7,\ a_3 = 7,\ a_8 = 7$

Algebra 10.1

Match

a_n is the number of decimal places in $(0.1)^n$ $\rightarrow (0.1)^3 = .001$ — $a_3 = 3$

a_n is the number of positive integers less than n^2 $\rightarrow 3^2 = 9$ — $a_3 = 8$

<u>Recursively defined sequences</u> - you must use previous terms to find the next term.

Find the third term in each of the following

① $a_1 = 2 \quad a_{k+1} = 3a_k + 5$

$a_2 = 3(2) + 5 = 11$

$a_3 = 3(11) + 5 = \boxed{38}$

② $a_1 = 5 \quad a_{k+1} = (k+1)a_k$

$a_2 = 2(5) = 10$

$a_3 = 3(10) = \boxed{30}$

③ $a_1 = 2 \quad a_{k+1} = (a_k)^{k+2}$

$k+1=2$ $\quad a_2 = (2)^3 = 8$

$k+2=3$ $\quad a_3 = (8)^4 = 8^4 = 4096$

④ $a_1 = 2 \quad a_2 = 3 \quad a_{k+1} = -2a_k + a_{k-1}$

$a_3 = -2a_2 + a_1 = -2(3) + 2 = -6 + 2 = -4$

$a_4 = -2a_3 + a_2 = -2(-4) + 3 = 8 + 3 = 11$

<u>Fibonacci Sequence</u> $1, 2, 3, 5, 13, 21, \ldots$

$a_1 = 1 \quad a_2 = 2 \quad a_{k+1} = a_k + a_{k-1}$

$a_3 = 2+1 = 3, \quad a_4 = 3+2 = 5, \quad a_5 = 5+3 = 8, \quad a_6 = 8+5 = 13$

Algebra 10.1

Sequence of Partial Sums

Find the first three terms of the sequence of partial sums for the following.

① $\{5-2n\}$

$a_1 = 5-2(1) = 3 \qquad s_1 = 3$

$a_2 = 5-2(2) = 1 \qquad s_2 = 3+1 = 4$

$a_3 = 5-2(3) = -1 \qquad s_3 = 3+1-1 = 3$

② $\{3+\frac{1}{2}n\}$

$a_1 = 3+\frac{1}{2}(1) = \frac{7}{2} \qquad s_1 = \frac{7}{2}$

$a_2 = 3+\frac{1}{2}(2) = 4 \qquad s_2 = \frac{7}{2}+4 = \frac{15}{2}$

$a_3 = 3+\frac{1}{2}(3) = \frac{9}{2} \qquad s_3 = \frac{7}{2}+4+\frac{9}{2} = \frac{24}{2} = 12$

③ $\{n\}$

$a_1 = 1 \qquad s_1 = 1$

$a_2 = 2 \qquad s_2 = 1+2 = 3$

$a_3 = 3 \qquad s_3 = 1+2+3 = 6$

$a_4 = 4 \qquad s_4 = 1+2+3+4 = 10$

$a_5 = 5 \qquad s_5 = 1+2+3+4+5 = 15$

$$\begin{array}{ccccc} & & & & 1 \\ & & & 2 & 3 \\ & & 4 & 5 & 6 \\ & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{array}$$

Algebra 10.1

Summations

$\sum_{K=1}^{5}$ means summation from $K=1$ to 5

① $\sum_{K=1}^{5} (2K-7) = -5-3-1+1+3 = -5$

② $\sum_{K=1}^{4} (K^2-5) = -4-1+4+11 = 10$

③ $\sum_{K=1}^{6} \left[1+(-1)^K\right] = 0+2+0+2+0+2 = 6$

④ $\sum_{K=0}^{4} K(K-2) = 0-1+0+3+8 = 10$

⑤ $\sum_{K=1}^{5} 3 = 3+3+3+3+3 = 15$

⑥ $\sum_{K=0}^{5} 3 = 3+3+3+3+3+3 = 18$

⑦ $\sum_{K=3}^{7} -4 = -4-4-4-4-4 = -20$

⑧ $\sum_{K=118}^{147} \frac{1}{3} = \frac{1}{3}(29+1) = \frac{1}{3}(30) = 10$

Algebra 10.2 Arithmatic Sequences

Ex. $-3, 4, 11, 18, \ldots$ $\underbrace{}_{+7}\underbrace{}_{+7}\underbrace{}_{+7}$ $d = 7$ $7n - 10$ — definition

$a_1 = 7(1) - 10 = -3$

Ex $53, 47, 41, 35, \ldots$ $d = 6$ $-6n + 59$

$a_1 = -6(1) + 59 = -6 + 59 = 53$

General Formula for finding the n^{th} term of an arithmatic sequence

$a_n = a_1 + (n-1)d$

Ex $10, -2, -14, -26, \ldots$

$a_1 = 10$ and $d = -12$

$a_n = 10 + (n-1)-12 = 10 - 12n + 12 = 22 - 12n$

Ex Find the 5^{th}, 10^{th}, and n^{th} term of the arithmatic sequence

$7, 11, 15, 19, \ldots$ $a_1 = 7$ and $d = 4$

$a_n = a_1 + (n-1)d$

$a_n = 7 + (n-1)4$

$a_n = 7 + 4n - 4$

$a_n = 3 + 4n$

$a_5 = 3 + 4(5) = 23$

$a_{10} = 3 + 4(10) = 43$

Ex Find the 8^{th}, 20^{th}, & n^{th} term of the arithmatic sequence

$-4, -0.5, 3, 6.5, \ldots$ $a_1 = -4$ and $d = 3.5$

$a_n = a_1 + (n-1)d = -4 + (n-1)3.5 = -4 + 3.5n - 3.5 = -7.5 + 3.5n$

$a_8 = -7.5 + 3.5(8) = -7.5 + 28 = 20.5$

$a_{20} = -7.5 + 3.5(20) = -7.5 + 70 = 62.5$

Algebra 10.2

EX Given $a_4 = 15$, $a_{11} = 43$ find a_n

one solution (slope) $d = \frac{43-15}{11-4} = \frac{28}{7} = 4$

if $a_4 = 15$, then $a_3 = 11$, $a_2 = 7$, $a_1 = 3$

another solution $a_4 = a_1 + (4-1)(4)$

$15 = a_1 + 12$

$a_1 = 3$

$a_n = 3 + (n-1)4 = 3 + 4n - 4 = 4n-1$

EX Given $a_8 = 41$, $a_9 = 46$, find a_n

$d = 5$

$a_8 = a_1 + (8-1)5$

$41 = a_1 + 35$

$a_1 = 6$

$a_n = 6 + (n-1)5$

$a_n = 6 + 5n - 5$

$a_n = 1 + 5n$

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

EX $a_n = 43 - 3n$ Find S_{26}

$S_{26} = \frac{26}{2}(a_1 + a_{26})$

$= 13((43-3(1)) + (43-3(26)))$

$= 13(40 - 35)$

$= \boxed{65}$

Algebra 10.2

EX Find the sum of the arithmetic sequence that satisfies the following conditions. $a_7 = -\frac{8}{3}, d = -\frac{2}{3}, n = 15$

$a_n = a_1 + (n-1)d$

$a_7 = a_1 + (7-1)(-\frac{2}{3})$

$-\frac{8}{3} = a_1 - \frac{12}{3}$

$a_1 = \frac{4}{3}$

$a_n = \frac{4}{3} + (n-1)(-\frac{2}{3})$

$a_n = \frac{4}{3} - \frac{2}{3}n + \frac{2}{3}$

$a_n = 2 - \frac{2}{3}n$

$S_{15} = \frac{15}{2}(a_1 + a_{15})$

$S_{15} = \frac{15}{2}(\frac{4}{3} - 8)$

$S_{15} = \frac{15}{2}(\frac{-20}{3})$

$S_{15} = 5(-10)$

$S_{15} = \boxed{-50}$

EX Express the sum in terms of summation notation

$11 + 16 + 21 + 26$

$d = 5 \quad a_1 = 11 \quad a_n = 5n + 6$ $\quad \boxed{\sum_{n=1}^{4} (5n+6)}$

EX express the sum in terms of summation notation

$-4, -9, -14, -19, -24$

$d = -5 \quad a_1 = -4 \quad a_n = -5n + 1$ $\quad \boxed{\sum_{n=1}^{5} (-5n+1)}$

Algebra 10.2

EX Express the sum in terms of summation notation

$1+3+5+\dots+73$

$d=2 \quad a_1=1 \quad a_n=2n-1$

$2n-1=73$

$2n=74$

$n=37$

$$\boxed{\sum_{n=1}^{37} 2n-1}$$

EX Express the sum in terms of summation notation

$\frac{3}{7}+\frac{6}{11}+\frac{9}{15}+\frac{12}{19}$

numerator: $d=3 \quad a_1=3 \quad a_n=3n$

denominator: $d=4 \quad a_1=7 \quad a_n=4n+3$

$$\boxed{\sum_{n=1}^{4} \frac{3n}{4n+3}}$$

EX $\sum_{n=5}^{19} 2n+3$

# of terms (19-5+1)

$$\frac{(15)(a_5+a_{19})}{2}=\frac{15(13+41)}{2}=\frac{15(54)}{2}=\frac{810}{2}=\boxed{405}$$

EX $\sum_{n=3}^{12} 5-2n$

$$\frac{(10)(a_3+a_{12})}{2}=\frac{10(-1-19)}{2}=\frac{10(-20)}{2}=\frac{-200}{2}=\boxed{-100}$$

iLrn 10.2 Part 3

Find the number of terms in the arithmatic sequence with the given conditions:

$a_1 = 8, \quad d = \frac{1}{4}, \quad S = -132$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{n}{2}(-8 + a_1 + (n-1)d)$

$-132 = \frac{n}{2}(-8 - 8 + (n-1)\frac{1}{4})$

$-132 = \frac{n}{2}(-16 + \frac{1}{4}n - \frac{1}{4})$

$-132 = \frac{n}{2}(-\frac{65}{8} + \frac{n}{8})$

$-1056 = -65n + n^2$

$n^2 - 65n + 1056 = 0$

$(n-32)(n-33) = 0$ try 32, 33

iLrn 10.2 Part 5

A contest will have five cash prizes totalling \$10,000, with a \$200 difference between successive prizes. Find the first prize.

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{n}{2}(a_1 + a_1 + (n-1)d)$

$10,000 = \frac{5}{2}(2a_1 + (5-1)(200))$

$10,000 = \frac{5}{2}(2a_1 + 800)$

⋮

$a_1 = 1600$

a_5 is first place, since these are in successive order

$a_5 = 1600 + (5-1)200$

$= 1600 + 800 = \boxed{\$2400}$

10.3 Geometric Sequences

Geometric Sequence: This sequence has a common ratio (r), or a value that is multiplied by one term to get the next term.

Ex: 2, -6, 18, -54, 162 … r = -3 Ex: $1, \frac{x}{3}, \frac{x^2}{9}, \frac{x^3}{27}, \ldots$ r = $\frac{x}{3}$

Ex: 99, 33, 11, $\frac{11}{3}$, … r = $\frac{1}{3}$ Ex: $1, \frac{-x}{3}, \frac{x^2}{9}, \frac{-x^3}{27}, \ldots$ r = $-\frac{x}{3}$

Ex: $\frac{2}{25}, \frac{2}{5}, 2, 10, 50, \ldots$ r = 5 Ex: $10, 10^{2x-1}, 10^{4x-3}, 10^{6x-5}, \ldots$ r = $\frac{10^{2x-1}}{10^1} = 10^{2x-2}$

Ex: 4, -6, 9, -13.5, … r = -1.5 $= -\frac{3}{2}$ Ex: $1, -\sqrt{3}, 3, -3\sqrt{3}, \ldots$ r = $-\sqrt{3}$

Let's derive a formula for finding the **nth term** of a geometric sequence by looking at an example.

Ex: 2, 6, 18, 54, 162, … r = 3 talk about the formula in reference to the first term

$2\cdot1, 2\cdot3, 2\cdot9, 2\cdot27, 2\cdot81, \ldots$

$2\cdot3^0, 2\cdot3^1, 2\cdot3^2, 2\cdot3^3, 2\cdot3^4, \ldots$ it looks like the formula will be $a_n = 2\cdot3^{n-1}$

In general, $\boxed{a_n = a_1 \cdot r^{n-1}}$ **(Memorize this!)**

Now, find a_n for the first 8 examples.

1. $a_n = 2\cdot(-3)^{n-1}$
2. $a_n = 99(\frac{1}{3})^{n-1}$
3. $a_n = \frac{2}{25}(5)^{n-1}$
4. $a_n = 4(-\frac{3}{2})^{n-1}$
5. $a_n = 1(\frac{x}{3})^{n-1}$
6. $a_n = 1(-\frac{x}{3})^{n-1}$
7. $a_n = 10(10^{2x-2})^{n-1}$
8. $a_n = 1(-\sqrt{3})^{n-1}$

If you are asked to find a later term, find a_n and plug in your specific value for n.

Ex: Find the 9th term of the geometric sequence 99, 33, 11, $\frac{11}{3}$, … $a_1 = 99$ $r = \frac{1}{3}$ $n = 9$

$a_9 = 99(\frac{1}{3})^8$

$a_9 = \frac{99}{6561} = \boxed{\frac{11}{729}}$

Ex: Find the 6th term of geometric sequence $1, \frac{-x}{3}, \frac{x^2}{9}, \frac{-x^3}{27}, \ldots$ $a_1 = 1$ $r = -\frac{x}{3}$ $n = 6$

$a_6 = 1(-\frac{x}{3})^5 = \boxed{\frac{-x^5}{243}}$

Ex: Find the 12th term of the geometric sequence whose first two terms are 4 and 12. $a_1 = 4$ $r = 3$ $n = 12$

$a_{12} = 4(3)^{11}$

$= 4(177147)$

$= \boxed{708588}$

Sometimes you need to find r, or a_1, or another term based upon two separated terms.

Ex: Find all possible values of r for a geometric sequence given $a_3 = 3$ and $a_6 = 81$ $\frac{81}{3} = 27 \Rightarrow \sqrt[3]{27} = 3$ $\boxed{r=3}$

Ex: Find all possible values of r for a geometric sequence given $a_7 = 5$ and $a_9 = 55$

$\frac{55}{5} = 11 \Rightarrow \sqrt[2]{11} = \boxed{\pm\sqrt{11}}$

Ex: The third term of a geometric sequence is 5, and the sixth term is -40. Find the 8th term.

$-\frac{40}{5} = -8 \Rightarrow \sqrt[3]{-8} = -2$ $r = -2$

$a_3 = a_1(-2)^{3-1}$

$5 = a_1(4) \Rightarrow a_1 = \frac{5}{4}$ $a_n = \frac{5}{4}(-2)^{n-1}$ $a_8 = \frac{5}{4}(-2)^7 = \boxed{-160}$

<u>**Sums**</u>: The sum of the first n terms is, $S_n = a_1\frac{1-r^n}{1-r}$

Ex: Find the sum: $\sum_{k=1}^{8} 2\cdot 3^k$ $a_1 = 2\cdot 3^1 = 6$ $r = 3$ $S_8 = 6\frac{1-3^8}{1-3} = 6\frac{-6560}{-2} = \boxed{19{,}680}$

Ex: Find the sum: $\sum_{k=1}^{10}(-2)^k$ $a_1 = (-2)^1 = -2$ $r = -2$ $S_{10} = -2\frac{1-(-2)^{10}}{1-(-2)} = -2\frac{-1023}{3} = \boxed{682}$

<u>Infinite Geometric Sequence</u>: The sum starts with the first term and keeps on going!

If $|r| < 1$, the the sum is $S = \frac{a_1}{1-r}$

Ex: Find the sum of the infinite geometric series: $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$ $r = \frac{1}{3}$ $S = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2\cdot\frac{3}{2} = \boxed{3}$

Ex: Find the sum of the infinite geometric series: $200 - 100 + 50 - 25 + \ldots$ $r = -\frac{1}{2}$ $S = \frac{200}{1+\frac{1}{2}} = 200\cdot\frac{2}{3} = \boxed{\frac{400}{3}}$

Ex: Find the sum of the infinite geometric series: $1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \ldots$ $r = \frac{3}{2}$

$|r|$ must be less than 1

not possible to find S

Ex: Find the sum of the infinite geometric series: $1.5 + 0.015 + 0.00015 + \ldots$ $r = .01$

$S = \frac{1.5}{1-.01} = \frac{1.5}{.99} = \frac{150}{99} = \boxed{\frac{50}{33}}$

You can also use and infinite geometric series to find the rational representation (fraction) of a repeating decimal.

Ex: Find the rational number represented by the repeating decimal: $0.\overline{3}$

$a_1 = .3$ $\quad r = .1$

$a_2 = .03$ $\quad S = \frac{.3}{1-.1} = \frac{.3}{.9} = \frac{3}{9} = \boxed{\frac{1}{3}}$

$a_3 = .003$

Ex: Find the rational number represented by the repeating decimal: $0.\overline{73}$

$a_1 = .73$ $\quad r = .01$

$a_2 = .0073$ $\quad S = \frac{.73}{1-.01} = \frac{.73}{.99} = \boxed{\frac{73}{99}}$

$a_3 = .000073$

Ex: Find the rational number represented by the repeating decimal: $15.\overline{2}$

$a_1 = .2$ $\quad r = .1$

$a_2 = .02$ $\quad S = \frac{.2}{1-.1} = \frac{.2}{.9} = \frac{2}{9}$ $\quad 15 + \frac{2}{9} = \boxed{\frac{137}{9}}$

$a_3 = .002$

Ex: Find the rational number represented by the repeating decimal: $2.4\overline{17}$

$a_1 = .017$ $\quad r = .01$

$a_2 = .00017$ $\quad S = \frac{.017}{1-.01} = \frac{.017}{.99} = \frac{17}{990}$ $\quad 2.4 + \frac{17}{990} = \boxed{\frac{2393}{990}}$

$a_3 = .0000017$

Applications

Ex: The yearly depreciation of a certain machine is 25% of it's value at the beginning of the year. If the original cost of the machine is $5000, what is it's value in 7 years?

$r = .75$ $\quad a_7 = 5000(.75)^{7-1}$

$a_1 = 5000$ $\quad = 5000(.75)^6$

$a_n = 5000(.25)^{n-1}$ $\quad = \boxed{\$889.89}$

Ex: A rubber ball is dropped from a height of 60 ft. If it rebounds approximately one-half the distance after each fall, use an infinite geometric series to approximate the total distance the ball travels.

↓60ft ↑30ft ↓30ft ↑15ft ↓15ft

going down: 60, 30, 15, ...

going up: 30, 15, ...

$a_n = 60(\frac{1}{2})^{n-1} \rightarrow S = 60 + 60 = 120$

$a_n = 30(\frac{1}{2})^{n-1} \rightarrow S = \frac{30}{1-\frac{1}{2}} = \frac{30}{\frac{1}{2}} = 30(2) = 60$

$\boxed{\text{total } 180\text{ft}}$

Algebra 10.5 The Binomial Theorem

Ex $(2x-3y)^3 = (2x-3y)(2x-3y)(2x-3y)$

$= (4x^2-12xy+9y^2)(2x-3y)$

$= 8x^3-24x^2y+18xy^2-12x^2y+36xy^2-27y^3$

$= 8x^3-36x^2y+54xy^2-27y^3$

Binomial Thrm

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Pascals Triangle

row 0	1
row 1	1 1
row 2	1 2 1
row 3	1 3 3 1
row 4	1 4 6 4 1
row 5	1 5 10 10 5 1
row 6	1 6 15 20 15 6 1

Ex: you have 6 friends but only 4 can be in your wedding. How many combinations can you have?

row 6 → entry 4 → $\binom{6}{4} = 15$

Ex Expand $(2x-3y)^3$ using Binomial Thrm.

$(2x-3y)^3 = \underline{1}\,(2x)^3(-3y)^0 + \underline{3}\,(2x)^2(-3y)^1 \underline{3}\,(2x)^1(-3y)^2 + \underline{1}\,(2x)^0(-3y)^3$

from Pascal's triangle

$= 8x^3-36x^2y+54x^2y^2-27y^3$

Alg 10.5

<u>EX</u> $\left(\frac{1}{2}c + d^3\right)^4$

14641 $= \underline{1}\left(\frac{1}{2}c\right)^4(d^3)^0 + \underline{4}\left(\frac{1}{2}c\right)^3(d^3)^1 + \underline{6}\left(\frac{1}{2}c\right)^2(d^3)^2 + \underline{4}\left(\frac{1}{2}c\right)^1(d^3)^3 + \underline{1}\left(\frac{1}{2}c\right)^0(d^3)^4$

$= \frac{1}{16}c^4 + 4\left(\frac{1}{8}\right)c^3 + 6\left(\frac{1}{4}\right)c^2(d^9) + 2cd^9 + d^{12}$

$= \frac{1}{16}c^4 + \frac{1}{2}c^3 + \frac{3}{2}c^2d^6 + 2cd^9 + d^{12}$

<u>EX</u> $(2x - y)^5$

5 10 10 5 1 $= 1(2x)^5 + 5(2x)^4(-y)^1 + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)^1(-y)^4 + 1(-y)^5$

$= 32x^5 + 5(16x^4)(-y) + 10(8x^3)(y^2) + 10(4x^2)(-y^3) + 10xy^4 - y^5$

$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

<u>EX</u> Find the term that has a^2b in $(3a-4b)^3$

$3(3a)^2(-4b)^1 = 3(9a^2)(-4b) = \boxed{-108a^2b}$