Guidelines for Solving Exponential or Logarithmic Equations

If you have a log equal to a number or a variable, change it to exponential form.

Ex: $\log_2 \frac{1}{16} = x$ $2^x = \frac{1}{16}$ $2^x = 2^{-4}$ $x = -4$

Ex: $\log_3 y = 4$ $3^4 = y$ $81 = y$

Ex: $\log_x 64 = 3$ $x^3 = 64$ $\sqrt[3]{x^3} = \sqrt[3]{64}$ $x = 4$

If you have a variable in your exponent, change it to logarithmic form.

Ex: $5^{2x-9} = 125$ $5^{2x-9} = 5^3$ $2x - 9 = 3$ $2x = 12$ $x = 6$

Ex: $2^{3x-1} = \frac{1}{2}$ $2^{3x-1} = 2^{-1}$ $3x - 1 = -1$ $3x = 0$ $x = 0$

If you have the same base on both sides of the equation, then simplify and set the exponents equal to each other.

Ex: $2^{x-3} = 2^{5x+13}$

$x - 3 = 5x + 13$

$-4x = 16$

$x = -4$

Ex: $(3^2)^x \cdot 3^{-x+1} = (3^{-1})^{3x-4} \cdot 3^2$

$2x - x + 1 = -3x + 4 + 2$

$4x = 5$

$x = 5/4$

If you don't have the same base on both sides of the equation, then try to get everything into the same base. Then, simplify and set the exponents equal to each other.

Ex: $(27)^x \cdot \left(\frac{1}{3}\right) = (9)^{x-4} \cdot 3^{-2}$

$3^{3x} \cdot 3^{-1} = 3^{2x-8} \cdot 3^{-2}$

$3x - 1 = 2x - 8 - 2$

$x = -9$

Ex: $4^{-2} \cdot (16^{2x})^3 = 8^{3x-2} \cdot \left(\frac{1}{32}\right)^{-x}$

$2^{-4} \cdot 2^{24x} = 2^{9x-6} \cdot 2^{5x}$

$-4 + 24x = 9x - 6 + 5x$

$10x = -2$

$x = -1/5$

If you don't have the same base on both sides of the equation, and you can't possibly get everything into the same base, then take the log or ln of both sides.

Ex: $7^{3x} = 11$

$\ln 7^{3x} = \ln 11$

$3x \ln 7 = \ln 11$

$3x = \frac{\ln 11}{\ln 7}$

$x = \frac{\ln 11}{3 \ln 7}$

Ex: $3^{x-2} = 14$

$\ln 3^{x-2} = \ln 14$

$(x-2)\ln 3 = \ln 14$

$x - 2 = \frac{\ln 14}{\ln 3}$

$x = \frac{\ln 14}{\ln 3} + 2$

If you have a radical (root) mixed in your equation, most of the time it helps to change it to a rational (fraction) exponent, and vice versa.

Ex: $\log_5 \sqrt[3]{5} = x$ $\log_5 5^{1/3} = x$, $5^x = 5^{1/3}$, $x = 1/3$

Ex: $\log_{16} x = \frac{3}{4}$ $16^{3/4} = x$, $(\sqrt[4]{16})^3 = x$, $2^3 = x$, $8 = x$

Ex: $\ln \sqrt[4]{e^3} = x$ $\ln e^{3/4} = x$, $\log_e e^{3/4} = x$, $e^x = e^{3/4}$, $x = 3/4$

If you have two logs of the same base set equal to one another, set the stuff in parenthesis (or that comes after the base) equal to each other.

Ex: $\log_2(x-3) = \log_2(9-5x)$
$x - 3 = 9 - 5x$
$6x = 12$
~~$x = 2$~~ no solution

Ex: $\ln\sqrt{x-3} = \ln 7$
$\sqrt{x-3} = 7$
$x - 3 = 49$
$x = 52$

Ex: $\log x^2 = \log(6-x)$
$x^2 = 6 - x$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x = 2, -3$

If you have several logs dancing around on both sides of the equation, try to get them all on one side and use your rules for logs.

Ex: $\log_4(x-2) = 1 - \log_4(x+2) + \log_4 3$
$\log_4(x-2) + \log_4(x+2) - \log_4 3 = 1$
$\log_4 \frac{(x-2)(x+2)}{3} = 1$
$\log_4 \left(\frac{x^2-4}{3}\right) = 1$
$4^1 = \frac{x^2-4}{3}$
$12 = x^2 - 4$
$x^2 = 16$
$x = \pm 4$ $x = 4$

Ex: $2\log_8 x + 4\log_8 2 = \log_8 x - 2$
$\log_8 x^2 + \log_8 2^4 - \log_8 x = -2$
$\log_8(16x^2) - \log_8 x = -2$
$\log_8\left(\frac{16x^2}{x}\right) = -2$
$\log_8 16x = -2$
$8^{-2} = 16x$
$\frac{1}{64} = 16x$
$\frac{1}{1024} = x$

If you have an equation that is quadratic in form, use substitution.

Ex: $5^x + 125 \cdot (5^{-x}) = 30$
$5^x + 125(\frac{1}{5^x}) = 30$
$5^{2x} + 125 = 30 \cdot 5^x$
$5^{2x} - 30 \cdot 5^x + 125 = 0$
let $y = 5^x$
$y^2 - 30y + 125 = 0$
$(y-25)(y-5) = 0$
$y = 5, 25$
$y = 5^x$
$5 = 5^x$ $25 = 5^x$
$x = 1$ $x = 2$
$x = 1, 2$

Ex: $e^{2x} + 2e^x - 15 = 0$ let $e^x = y$
$y^2 + 2y - 15 = 0$
$(y+5)(y-3) = 0$
$y = -5$ $y = 3$
$e^x = -5$, $\log_e -5 = x$, ~~$e^x = -5$~~
$e^x = 3$, $\log_e 3 = x$, $x = \ln 3$

And sometimes you just need to use some good ole factoring.

Ex: $x^2 e^{3x} = 7xe^{3x}$
$x^2 e^{3x} - 7xe^{3x} = 0$
$xe^{3x}(x-7) = 0$
$xe^{3x} = 0$ $x - 7 = 0$
$x = 0$ $x = 7$

Ex: $(\log x)^4 = 27\log x$
$(\log x)^4 - 27\log x = 0$
$\log x((\log x)^3 - 27) = 0$
$\log x = 0$, $10^0 = x$, $x = 1$
$(\log x)^3 - 27 = 0$, $(\log x)^3 = 27$, $\log x = 3$, $10^3 = x$ $x = 1000$
$x = 1, 1000$