Summary of Inequalities

Absolute Value Inequalities

Isolate the absolute value first, and then follow either step one or two:

1. Less than symbol: less than**d**, set up an "and" compound inequality and solve.
2. Greater than symbol: great**or** than, set up an "or" compound inequality and solve.

Ex 1: Solve $|3x-2|-7<0$

$|3x-2|<7$

$-7<3x-2<7$

$-5<3x<9$

$-\frac{5}{3}<x<3$

$(-\frac{5}{3}, 3)$

Ex 2: Solve $|-2x+9|\geq 1$

$-2x+9\leq -1$ or $-2x+9\geq 1$

$-2x\leq -10$ $\quad -2x\geq -8$

$x\geq 5$ or $x\leq 4$

$(-\infty, 4]\cup[5, \infty)$

Ex 3: Solve $|6x+8|<-1$

No solution $\emptyset$

Ex 4: Solve $|6x+8|>-1$

Same as $|6x+8|\geq 0$

$\mathbb{R}$ or $(-\infty, \infty)$

Linear Inequalities (highest power of x is 1)

Get x by itself on the left hand side. Your answer should be a single interval.

Ex 5: Solve $-3x+2>x+10$

$-4x>8$

$x<-2$

$(-\infty, -2)$

Ex 6: Solve $(2x-3)(5x+1)\leq 10x^2-x$

$10x^2+2x-15x-3\leq 10x^2-x$

$-13x-3\leq -x$

$-12x-3\leq 0$

$-12x\leq 3$

$x\geq -\frac{3}{12}$

$x\geq -\frac{1}{4}$ $\quad [-\frac{1}{4}, \infty)$

Inequalities of degree 2 or more

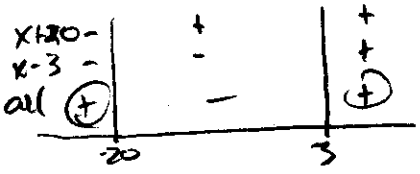
Get all terms on the left side (zero on the right), factor the left side, and set up a sign diagram! This is the only type of inequality you use a sign diagram for. You must be careful not to include values that make any denominators zero, and be sure to include values that make the numerator zero when you have $\leq$, or $\geq$.

Ex 7: Solve $x^2+12x \geq -5x+60$

$x^2+17x-60 \geq 0$

$(x+20)(x-3) \geq 0$

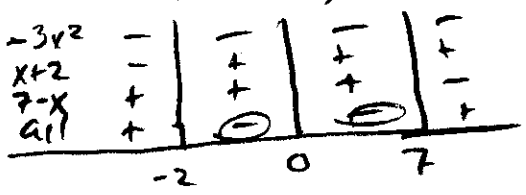
test values are $-20, 3$



$(-\infty, -20] \cup [3, \infty)$

Ex 8: Solve $-3x^2(x+2)(7-x) \leq 0$

test values are $0, -2, 7$



$[-2, 7]$

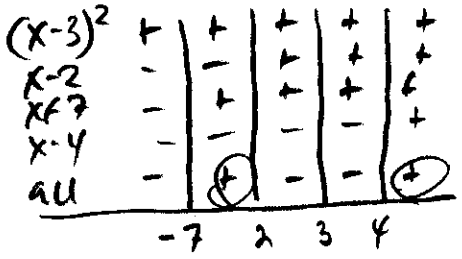
$x \neq -7, 4$

Ex 9: Solve $\dfrac{(x-3)(x^2-5x+6)}{x^2+3x-28} \geq 0$

$x = 3, 2$

$\dfrac{(x-3)(x-3)(x-2)}{(x+7)(x-4)} \geq 0$

test values are $-7, 2, 3, 4$



$(-7, 2] \cup \{3\} \cup (4, \infty)$

Ex 10: Solve $\dfrac{(x-3)^2(x-2)}{(x+7)(x-4)} > 0$

$x \neq -7, 4$ same as last example

$(-7, 2) \cup (4, \infty)$

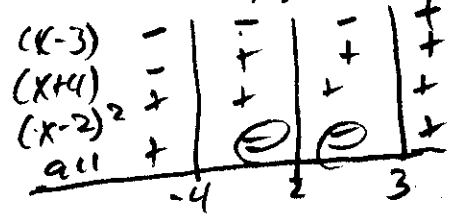
Ex 11: Solve $\dfrac{-(x+4)(3-x)}{(x-2)^2} \leq 0$

$x \neq 2$

$x = 3, 4$

$\dfrac{(x-3)(x+4)}{(x-2)^2} \leq 0$

test values $2, 3, -4$

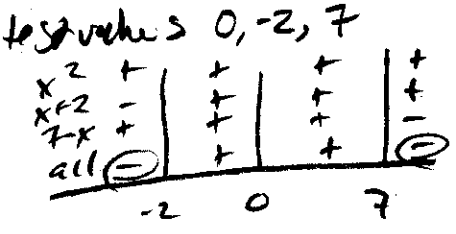


$-4 \Rightarrow 3$

$[-4, 2) \cup (2, 3]$

Ex 12: Solve $x^2(x+2)(7-x) \leq 0$

test values $0, -2, 7$



$(-\infty, -2] \cup \{0\} \cup [7, \infty)$

Due Thurs

Bonus (2pts): The number of miles M that a certain compact car can travel on 1 gallon of gasoline is related to its speed v (in mi/hr) by: $M = -\dfrac{1}{30}v^2 + \dfrac{5}{2}v$, for $0 < v < 70$. For what speeds will M be at least 45?

Luke Spence

x2

Bonus (Summary of Inequalities)

The number of miles M that a certain compact car can travel on 1 gallon of gasoline is related to it's speed v in mph by $M = -\frac{1}{30}v^2 + \frac{5}{2}v$, for $0 < v < 70$
For what speed will M be at least 45?

$-\frac{1}{30}v^2 + \frac{5}{2}v = M$

$-\frac{1}{30}v^2 + \frac{5}{2}v \leq 45$

$30(-\frac{1}{30}v^2 + \frac{5}{2}v) \leq 45(30)$

$-v^2 + 75v \leq 1350$

$-v^2 + 75v - 1350 \leq 0$

$v^2 - 75v + 1350 \geq 0$

$(v-30)(v-45) \geq 0$

test values 30, 45

$(v-30)$	$-$	$+$	$+$
$(v-45)$	$-$	$-$	$+$
all	$+$	$\ominus$	$+$
		30	45

$[30, 45]$