Trig 10.4 Mathmatical Induction

<u>Review of Sequences</u>

$\{1, 1/2, 1/3, 1/4, 1/5, \ldots 1/n, \ldots\}$

$\{2, 4, 6, 8, 10, \ldots 2n, \ldots\}$

$\{1, 4, 9, 16, 25, \ldots n^2, \ldots\}$

$\{1, 3, 5, 7, 9, \ldots 2n-1, \ldots\}$

sequences $\begin{cases} \text{① index (1st term)} \\ \text{② expression of the } n^{th} \text{ term} \end{cases}$

$\{2, 7, 12, \ldots (5n-3), \ldots\}$

$a_2 = 7$

$a_n = 5n-3$ (a_n tells a relationship between its index & its value)

$\{2, 4, 6, 8, \ldots 2n\}$

The sum of the first n terms, usually has a rule in it.

$2+4+6+8 \ldots + 2n = n(n+1)$

sum of first 5, $2+4+6+8+10 = 5(6) = 30$

<u>Dominoes</u> - conditions that a domino will fall

① Push 1st domino

② dominoes are properly spaced

$1+2+3+4+\ldots+n = \frac{n(n+1)}{2}$ result we want

need 2 conditions $\begin{cases} \text{① check 1st term} \\ \text{② make sure } a_2 \ldots a_n \text{ has same properties} \end{cases}$

① $LS = 1$

$RS = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = 1$

$LS = RS$ ✓

② $n=4$ $LS = 1+2+3+4 = 10$

$RS = \frac{4(4+1)}{2} = \frac{20}{2} = 10$

$LS = RS$ ✓

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Ex Prove when $n=1$, the following statements hold

① $1+3+5+\ldots(2n-1)=n^2$

$n=1$ $LS=1$

$RS=n^2=1^2=1$

$LS=RS$ ✓

$n=3$ $LS=1+3+5=9$

$RS=3^2=9$

$LS=RS$ ✓

Assume when $n=k$ the equation holds

$a_n=2n-1$

$a_{n+1}=2(n+1)-1$

$a_3=2(3)-1=5$

② $2+7+12+\ldots+(5n-3)=\frac{1}{2}n(5n-1)$

$n=1$ $LS=2$

$RS=\frac{1}{2}n(5n-1)=\frac{1}{2}(1)(5(1)-1)=2$

$LS=RS$ ✓

$n=3$ $LS=2+7+12=21$

$RS=\frac{1}{2}(3)(5(3)-1)=\frac{3}{2}(14)=21$

$LS=RS$ ✓

term	value
a_1	$=2$
a_2	$=7$
a_n	$=5n-3$
a_k	$=5k-3$
a_{k-1}	$=5(k-1)-3=5k-8$

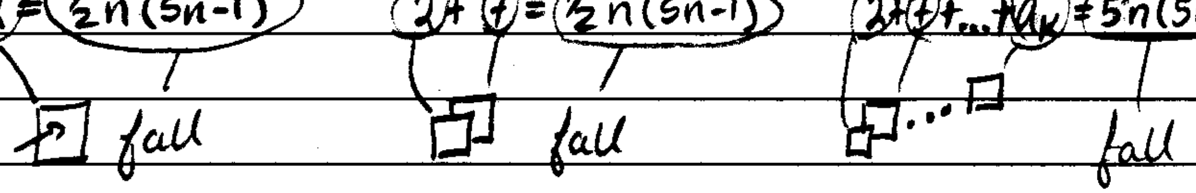
$n=\quad 1 \quad 2 \quad 3 \quad \ldots \quad n$

$a_n=2+7+12+\ldots+(5n-3)=\frac{1}{2}n(5n-1)$

Imagine $a_1, a_2, \ldots a_n$ are dominos, the result $\frac{1}{2}n(5n-1)$ is similiar to the dominos falling down.

$n=1$ $\quad\quad$ $n=2$ $\quad\quad$ $n=k$

$2=\frac{1}{2}n(5n-1)$ $\quad\quad$ $2+7=\frac{1}{2}n(5n-1)$ $\quad\quad$ $2+7+\ldots+a_k=5n(5n-1)$



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Induction

Prove $2+7+12+\ldots+(5n-3)=\frac{1}{2}n(5n-1)$

① $n=1$ $LS=2$

$RS=\frac{1}{2}(1)(5(1)-1)=\frac{1}{2}(4)=2$

$LS=RS$ ✓

② assume $n=k$ the equation holds

$2+7+12+\ldots+(5k-3)=\frac{1}{2}k(5k-1)$

③ (We need to prove when $n=k+1$, the equation also holds)

$LS=\underbrace{2+7+12+\ldots+(5k-3)}+5(k+1)-3$

$=\frac{1}{2}k(k-1) \quad +5(k+1)-3$

$=\frac{5}{2}k^2-\frac{1}{2}k+5k+5-3$

$=\frac{5}{2}k^2+\frac{9}{2}k+2$

$RS=\frac{1}{2}(k+1)(5(k+1)-1)$

$=\left(\frac{1}{2}k+\frac{1}{2}\right)(5k+4)$

$=\frac{5}{2}k^2+\frac{5}{2}k+2k+2$

$=\frac{5}{2}k^2+\frac{9}{2}k+2$

$LS=RS$ ✓

④ SO

$2+7+\ldots+(5n-3)=\frac{n}{2}(5n-1)$ for all $n\geq 1$

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Prove $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$a_n = n^2 \quad a_K = K^2 \quad a_{K+1} = (K+1)^2$

① $n=1$ $LS = 1^2 = 1$

$RS = \frac{1(1+1)(2\cdot 1+1)}{6} = \frac{2\cdot 3}{6} = \frac{6}{6} = 1$

$LS = RS$ ✓

② Assume $n=K$

$1^2+2^2+3^2+\dots+K^2 = \frac{K(K+1)(2K+1)}{6}$

③ $n = K+1$

$LS = \underbrace{1^2+2^2+3^2+\dots+K^2}+(K+1)^2$

$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= \frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

$= \frac{(K+1)[K(2K+1)+6(K+1)]}{6}$

$= \frac{(K+1)(2K^2+K+6K+6)}{6}$

$= \frac{(K+1)(2K^2+7K+6)}{6}$

$= \frac{(K+1)(K+2)(2K+3)}{6}$

$RS = \frac{(K+1)((K+1)+1)(2(K+1)+1)}{6}$

$= \frac{(K+1)(K+2)(2K+3)}{6}$

$LS = RS$ ✓

④ So

$1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \geq 1$

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Prove $2+4+6+\dots+2n = n(n+1)$

$a_n = 2n \quad a_K = 2K \quad a_{K+1} = 2(K+1)$

① $n=1$ $LS = 2$

$RS = 1(1+1) = 2$

$LS = RS$ ✓

② assume $n=K$

$2+4+6+\dots+2K = K(K+1)$

③ $n = K+1$

$LS = \underbrace{2+4+6+\dots+2K}_{} + 2(K+1)$

$= K(K+1) + 2(K+1)$

$= (K+2)(K+1)$

$RS = (K+1)((K+1)+1)$

$= (K+1)(K+2)$

$LS = RS$ ✓

④ So

$2+4+6+\dots+2n = n(n+1)$ for all $n \geq 1$