

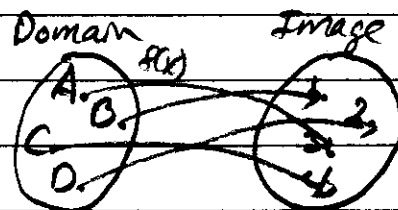
## Aug 3.8 Inverse Functions

### Two Concepts

- 1) 1-to-1 functions - given a function you can prove it's a 1-to-1 function
- 2) Inverse functions - given a function, find it's inverse function

### 1-to-1 Functions - $f(x)$

when  $a \neq b \Rightarrow f(a) \neq f(b)$  ) equivalent  
or  $f(a) = f(b) \Rightarrow a = b$



EX1. Prove  $f(x) = \frac{1}{x}$  is 1-1 function

assume  $f(a) = \frac{1}{a}$   $f(b) = \frac{1}{b}$

$$f(a) = f(b) \Rightarrow \frac{1}{a} = \frac{1}{b} \Rightarrow a = b$$

so  $f(x)$  is 1-to-1

EX2. Is  $f(x) = x^2 - 9$  a 1-1 function? why?

assume  $f(a) = f(b)$

$$f(a) = a^2 - 9 \quad f(b) = b^2 - 9$$

$$f(a) = f(b) \Rightarrow a^2 - 9 = b^2 - 9 \Rightarrow a^2 = b^2 \Rightarrow a = \pm b$$

$a \neq b$  do not 1-1

EX3.  $f(x) = \sqrt[9]{x}$  is  $f(x)$  1-1? why?

assume  $f(a) = f(b)$

$$f(a) = \sqrt[9]{a} \quad f(b) = \sqrt[9]{b}$$

$$f(a) = f(b) \Rightarrow \sqrt[9]{a} = \sqrt[9]{b} \Rightarrow (\sqrt[9]{a})^9 = (\sqrt[9]{b})^9 \Rightarrow a = b$$

$a = b$  yes is 1-to-1 function

## Trig 3.8

### Inverse Functions - $f^{-1}(x)$

Let  $f$  be 1-1, with domain  $D$  and range  $R$ .

The inverse function of  $f$  is 1-1 function, with domain  $R$ , range  $D$

$$\text{if } y = f(x) \Leftrightarrow x = f^{-1}(y)$$

EX1.  $f(x) = 3x + 5$  Find its inverse function  $f^{-1}(x)$

Sol.  $f(x) = 3x + 5$

$$y = 3x + 5 \Rightarrow \frac{y-5}{3} = x \quad f^{-1}(x) = \frac{x-5}{3}$$

EX2 Find  $f^{-1}(x)$  for  $f(x) = \frac{1}{3x-2}$

Sol.  $y = \frac{1}{3x-2}$   $3x-2 = \frac{1}{y}$   $3x = 2 + \frac{1}{y}$   $3x = \frac{2y+1}{y}$   $x = \frac{2y+1}{3y}$   
 $f^{-1}(y) = \frac{2y+1}{3y}$   $f^{-1}(x) = \frac{2x+1}{3x}$

EX3  $f(x) = 2 - 3x^2$   $x \leq 0$   $f^{-1}(x) = ?$

$$y = 2 - 3x^2 \quad -3x^2 = y - 2 \quad x^2 = \frac{y-2}{-3} \quad x = -\sqrt{\frac{y-2}{-3}} \quad f^{-1}(x) = -\sqrt{\frac{x-2}{-3}}$$

EX4  $f(x) = 2x^3 - 5$  find  $f^{-1}(x) = ?$

$$y = 2x^3 - 5$$

$$y + 5 = 2x^3$$

$$x^3 = \frac{y+5}{2}$$

$$x = \sqrt[3]{\frac{y+5}{2}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$$