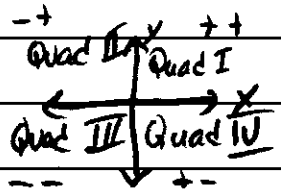
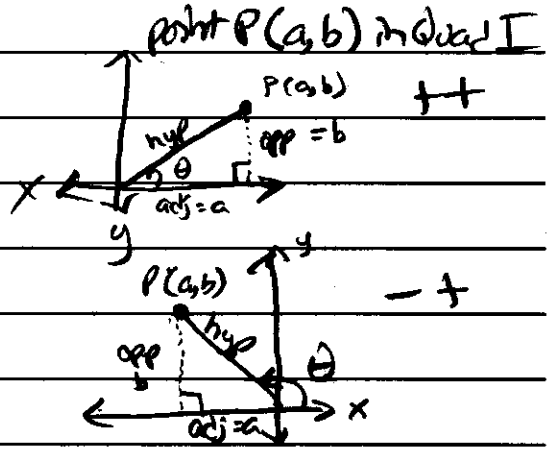
Trig 6.2 Trigonometry Functions of Any θs 1/12/2007

1. Definitions & Properties of 6 Trig functions w/problems
2. Examples

1. a Definition

$\sin\theta = \frac{opp}{hyp}$	
$\cos\theta = \frac{adj}{hyp}$	
$\tan\theta = \frac{opp}{adj}$	
$\cot\theta = \frac{adj}{opp}$	
$\sec\theta = \frac{hyp}{adj}$	
$\csc\theta = \frac{hyp}{opp}$	





b. Properties

① Reciprical relationships

a) $\tan\theta = \frac{1}{\cot\theta}$ $\left(\cot\theta = \frac{1}{\tan\theta}\right)$

b) $\sec\theta = \frac{1}{\cos\theta}$ $\left(\cos\theta = \frac{1}{\sec\theta}\right)$

c) $\sin\theta = \frac{1}{\csc\theta}$ $\left(\csc\theta = \frac{1}{\sin\theta}\right)$

② tangent & cotangent relationship w/ sine & cosine

$$\tan\theta = \frac{opp}{adj} = \frac{opp}{adj}\cdot\frac{\frac{1}{hyp}}{\frac{1}{hyp}} = \frac{opp/hyp}{adj/hyp} = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{adj}{opp} = \frac{adj}{opp}\cdot\frac{\frac{1}{hyp}}{\frac{1}{hyp}} = \frac{adj/hyp}{opp/hyp} = \frac{\cos\theta}{\sin\theta}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$

③ Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot\theta = \csc^2\theta$

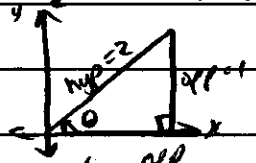
Trig 6.2

Given 1 trig function value, find the other 5 trig function values

Steps

1. Draw a right triangle
2. From the given condition, get two sides
3. Use pythagoreans identities to get 3rd side
4. Use def. of trig func. Find $\sin\theta = ?$ $\cos\theta = ?$
5. Use properties, find the other 4 trig functions

Ex 1 $\sin\theta = \frac{1}{2}$ θ is in Quad. I, find the other 5 trig func. values

Sol. ① 

Right Triangle

$adj^2 + opp^2 = hyp^2$

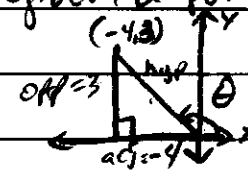
② $\sin\theta = \frac{1}{2} = \frac{opp}{hyp}$

③ $adj^2 + opp^2 = hyp^2$ $adj^2 + 1^2 = 2^2$ $adj^2 = 3$ $adj = \sqrt{3}$

④ $\sin\theta = \frac{1}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\tan\theta = \frac{1}{\sqrt{3}}$

$\csc\theta = 2$ $\sec\theta = \frac{2}{\sqrt{3}}$ $\cot\theta = \sqrt{3}$

Ex 2 given a point $P(-4,3)$, find 6 trig values

① 

② $opp = 3$

$adj = -4$

③ $hyp^2 = 3^2 + 4^2 \Rightarrow hyp^2 = 25$ $hyp = 5$ hyp is always pos. None

④ $\sin\theta = \frac{3}{5}$ $\cos\theta = -\frac{4}{5}$ $\tan = -\frac{3}{4}$

$\csc\theta = \frac{5}{3}$ $\sec\theta = \frac{5}{-4}$ $\cot = -\frac{4}{3}$

Trig 6.2

Fundamental Identities

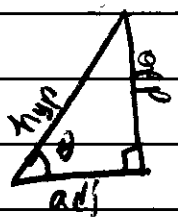
$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$

** **Verify Identities**

$\sin X \cdot \sec X = \tan X$

$\frac{\sec\beta \cdot \cos\beta}{1+\tan^2\beta} = \cos^2\beta$



Two Ways to Verify - Basic / Advanced

Basic Way ① left side, copy then represent each term w/ sin or cos & simplify.

② Right side, repeat

③ Find left side = Right side

Advanced Way - use Fundamental Identities

Basic Way

$\sin X \cdot \sec X = \tan X$

Step 1: $LS = \sin X \cdot \sec X = \sin X \cdot \frac{1}{\cos X} = \frac{\sin X}{\cos X}$

Step 2: $RS = \tan X = \frac{\sin X}{\cos X}$

Step 3: $LS = RS$ ✓

$\frac{\sec\beta\cos\beta}{1+\tan^2\beta} = \cos^2\beta$

Step 1: $LS = \frac{\sec\beta\cos\beta}{1+\tan^2\beta} = \frac{\frac{1}{\cos\alpha}\cdot\cos\beta}{1+\left(\frac{\sin\beta}{\cos\beta}\right)^2} = \frac{1}{1+\frac{\sin^2\beta}{\cos^2\beta}} = \frac{1}{\frac{\cos^2\beta}{\cos^2\beta}+\frac{\sin^2\beta}{\cos^2\beta}} = \frac{1}{\frac{\cos^2\beta+\sin^2\beta}{\cos^2\beta}} = \frac{\cos^2\beta}{\cos^2\beta+\sin^2\beta}$

$= \frac{\cos^2\beta}{1} = \cos^2\beta$

Step 2: $RS = \cos^2\beta$

Step 3: $LS = RS$ ✓

Advanced Way

$\frac{\sec\beta\cos\beta}{1+\tan^2\beta} = \cos^2\beta$

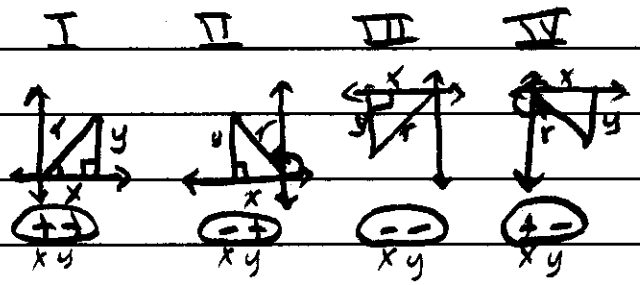
Step 1: $LS = \frac{\sec\beta\cos\beta}{1+\tan^2\beta} = \frac{\sec\beta\cos\beta}{\sec^2\beta} = \frac{\cos\beta}{\sec\beta} = \frac{\cos\beta}{\frac{1}{\cos\beta}} = \cos\beta\cos\beta = \cos^2\beta$

Step 2: $RS =$

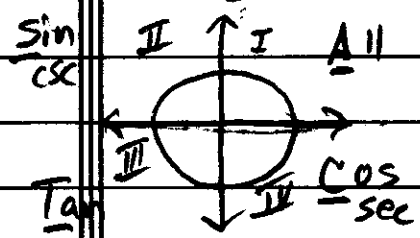
Step 3:

Trig 6.2

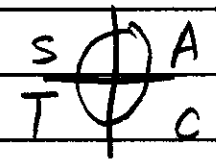
	I	II	III	IV
$\sin\theta = \frac{y}{r}$	+	+	−	−
$\cos\theta = \frac{x}{r}$	+	−	−	+
$\tan\theta = \frac{y}{x}$	+	−	+	−
$\cot\theta = \frac{x}{y}$	+	−	+	−
$\sec\theta = \frac{r}{x}$	+	−	−	+
$\csc\theta = \frac{r}{y}$	+	+	−	−



r (hypotenuse) is always positive



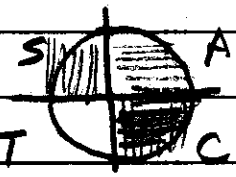
Remember Cast



Problem: Given $\tan\theta < 0$ & $\cos\theta > 0$

Which Quadrant is θ in? IV

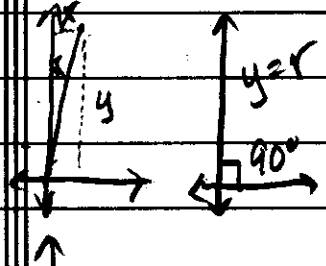
so $\theta \in$ QIV



Simplify $\frac{\cot^2\alpha - 4}{\cot^2\alpha - \cot\alpha - 6} = \frac{x^2-4}{x^2-x-6} = \frac{(x-2)(x+2)}{(x-3)(x+2)} = \frac{x-2}{x-3} = \frac{\cot\alpha - 2}{\cot\alpha - 3}$

let $x = \cot\alpha$

Find 6 Trig Function values when $\theta = 90°$ 0° 270°



as angle approaches 90° $y = r$, and $x = 0$

$\sin 90 = \frac{y}{r} = \frac{r}{r} = 1$

$\cos 90 = \frac{x}{r} = \frac{0}{r} = 0$

$\tan 90 = \frac{y}{x} = \frac{r}{0} = \text{ND}$

$\sec 90 = \frac{1}{\cos 90°} = \text{ND}$

$\csc 90 = \frac{1}{\sin 90} = 1$

90° $y = r$

0° $y = 0$, $x = r$

270° $y = -r$, $x = 0$