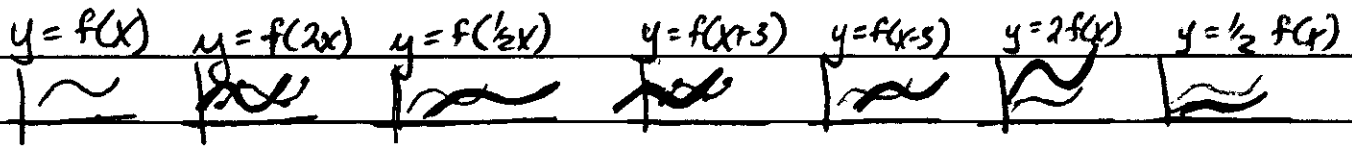


Trig 6.5 Trigonometric Graphs



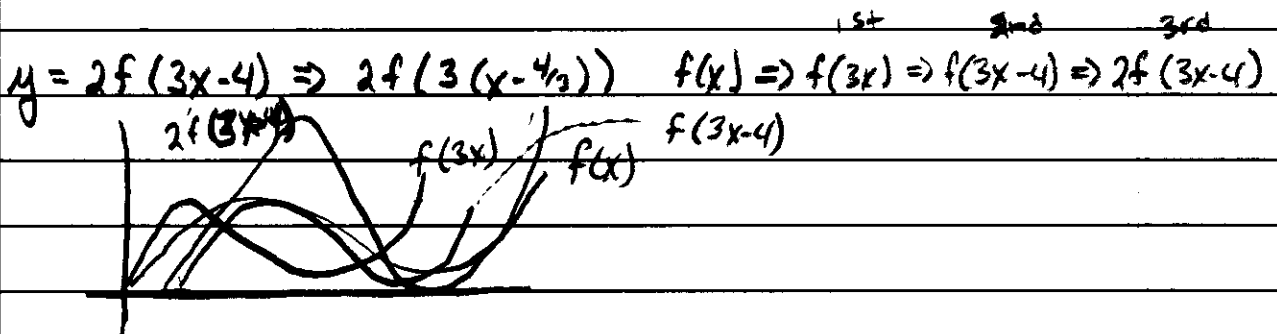
Shift x-axis $y = f(x+c)$

Stretch x-axis $y = f(bx)$

Stretch y-axis $y = a f(x)$

$$y = a f(bx+c)$$

\uparrow \uparrow \rightarrow shift $\begin{cases} c > 0 & \text{shift left } c \text{ units} \\ c < 0 & \text{shift right } c \text{ units} \end{cases}$
 \rightarrow shrink/stretch on x axis $\begin{cases} |b| > 1 & \text{shrink } |b| \text{ times} \\ |b| < 1 & \text{stretch } |b| \text{ times} \end{cases}$
 \rightarrow shrink/stretch on y axis $\begin{cases} |a| > 1 & \text{stretch } |a| \text{ times} \\ |a| < 1 & \text{shrink } |a| \text{ times} \end{cases}$



$$y = f(x) = a \sin(bx+c)$$

e.g. $y = 2 \sin(5x-7)$

1. Amplitude = a

2. Period = $\sin, \cos \frac{2\pi}{|b|}, \tan \frac{\pi}{|b|}$

3. Phase (shift) = $-\frac{c}{b}$

4. Interval = $\sin, \cos 0 \leq bx+c \leq 2\pi, \tan -\frac{\pi}{2} < bx+c < \frac{\pi}{2}$

Trig 6.5

EX1. Find amplitude, period, phase & interval of $y = 4 \cos(x - \frac{\pi}{4})$

$$a = 4 \quad b = 1 \quad c = -\frac{\pi}{4}$$

$$\text{Amplitude} = a = 4$$

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi$$

$$\text{Phase} = -\frac{c}{b} = -\frac{-\frac{\pi}{4}}{1} = \frac{\pi}{4}$$

$$\text{Interval } 0 \leq bx + c \leq 2\pi \Rightarrow 0 \leq x - \frac{\pi}{4} \leq 2\pi \Rightarrow \frac{\pi}{4} \leq x \leq \frac{9\pi}{4} \left[\frac{\pi}{4}, \frac{9\pi}{4} \right]$$

EX2. Find amplitude, period, phase & interval of $y = -\frac{1}{2} \sin(-\frac{1}{3}x - \frac{1}{4})$

$$a = -\frac{1}{2} \quad b = -\frac{1}{3} \quad c = -\frac{1}{4}$$

$$\text{Amplitude} = a = -\frac{1}{2}$$

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{\frac{1}{3}} = 6\pi$$

$$\text{Phase} = -\frac{c}{b} = -\frac{-\frac{1}{4}}{-\frac{1}{3}} = -\frac{3}{4}$$

$$\text{Interval } 0 \leq -\frac{1}{3}x - \frac{1}{4} \leq 2\pi$$

$$\frac{1}{4} \leq -\frac{1}{3}x \leq 2\pi + \frac{1}{4}$$

$$-\frac{3}{4} \geq x \geq 6\pi - \frac{3}{4} \quad \left[-6\pi - \frac{3}{4}, -\frac{3}{4} \right]$$

EX3 Given amp = 2, period = 2π , phase = $-\pi$, find the sine function

Sol. We know the sine function in general form $y = a \sin(bx + c)$

we only need to find a, b, c

$$\text{Amplitude} = 2 \quad a = 2$$

$$\text{Period} = 2\pi \Rightarrow \frac{2\pi}{|b|} = 2\pi \quad |b| = 1$$

$$\text{Phase} = -\pi \quad -\frac{c}{b} = -\pi \Rightarrow -\frac{c}{1} = -\pi \Rightarrow c = \pi$$

$$\Rightarrow -\frac{c}{1} = -\pi \Rightarrow c = -\pi$$

$$y = 2 \sin(x + \pi)$$

or

$$y = 2 \sin(-x - \pi)$$