Trig 7.1 Verifying Trig Identities

Two ways to verify

1) Traditional Way - transform LS & transform RS $\Rightarrow$ LS = RS
2) Special Way - for special types of problems

Ex Prove $\csc\theta - \sin\theta = \cot\theta\cos\theta$

$LS = \csc\theta - \sin\theta$

$= \frac{1}{\sin\theta} - \sin\theta$

$= \frac{1-\sin^2\theta}{\sin\theta}$ $(\sin^2\theta + \cos^2\theta = 1)$

$= \frac{\cos^2\theta}{\sin\theta}$

$RS = \cot\theta\cos\theta$

$= \frac{\cos\theta}{\sin\theta}\cdot\cos\theta$

$= \frac{\cos^2\theta}{\sin\theta}$

$LS = RS$ ✓

Special Problem

$\frac{x^2}{\sqrt{a^2-x^2}} = a\tan\theta\sin\theta$ $\boxed{a>0,\ a \text{ is a real } \#,\ -90° \le \theta \le 90°}$

$LS = \frac{x^2}{\sqrt{a^2-x^2}}$ let $x = a\sin\theta$ $x \in (-\infty, \infty)$ $-\infty \le a\sin\theta \le \infty$

$= \frac{(a\sin\theta)^2}{\sqrt{a^2-(a\sin\theta)^2}}$

$= \frac{a^2\sin^2\theta}{\sqrt{a^2-a^2\sin^2\theta}} = \frac{a^2\sin^2\theta}{\sqrt{a^2(1-\sin^2\theta)}} = \frac{a^2\sin^2\theta}{\sqrt{a^2\cos^2\theta}} = \frac{a^2\sin^2\theta}{|a\cos\theta|} = \frac{a\sin^2\theta}{|\cos\theta|}$

cos is positive

$= \frac{a\sin^2\theta}{\cos\theta}$

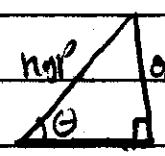
$RS = a\tan\theta\sin\theta$

$= a\frac{\sin\theta}{\cos\theta}\cdot\sin\theta$

$= \frac{a\sin^2\theta}{\cos\theta}$

$LS = RS$ ✓

$x = 10009.7231$
$\parallel$
$a\sin\theta$



sin is always decimal less than 1

Trig 7.1

Make the trigonometric substitution to simplify the following expression, by letting $x = a\sin\theta$ for $-90 \le \theta \le 90$ and $a > 0$

Make a trig substitution for $x = a\tan\theta$ w/ $a > 0$ $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sqrt{x^2 + a^2}$

$\sqrt{(a\tan\theta)^2 + a^2}$

$\sqrt{a^2\tan^2\theta + a^2}$

$\sqrt{a^2(\tan^2\theta + 1)}$

$\sqrt{a^2(\sec^2\theta)}$

$\frac{\cos^2\theta}{a^2}$

1st remove fraction then prove

1) $(\sec t + \tan t)^2 = \frac{1+\sin t}{1-\sin t} \Rightarrow (\sec t + \tan t)^2(1-\sin t) = 1+\sin t$

2.) $\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2\cos^2\alpha \Rightarrow 1+\cos\alpha + 1-\cos\alpha = 2\cos^2\alpha + 2\cos^2\alpha + 2\cos\alpha$

$2 = 4\cos^2\alpha$

Verify Identity

$\cos^2 3x - \sin^2 3x = \cos 6x$

let $3x = \theta$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$ Double angle formula $(\cos^2 u - \sin^2 u = \cos(2u))$

$LS = \cos^2\theta - \sin^2\theta$

$= \cos 2\theta$

$RS = \cos 2\theta$ ✓