Trig 7.4 Multiple Angle Functions

1) Double Angle Formula

$$\boxed{\begin{aligned} \sin(2u) &= 2\sin u\cos u \\ \cos(2u) &= \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1 \end{aligned}}$$

$$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$$

EX - Find the exact value of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$ for $\cos\theta = 3/5$ $0° < \theta < 90°$

sol - $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{4}{5}\cdot\frac{3}{5} = \frac{24}{5}$

$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{3}{5}\right)^2 - 1 = \frac{18}{25} - 1 = -\frac{7}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot 4/3}{1-(4/3)^2} = \frac{8/3}{-7/9} = \frac{8}{3}\cdot\left(-\frac{9}{7}\right) = -\frac{24}{7}$

EX $90° < \theta < 180°$ $\sec\theta = -3$ Find $\sin 2\theta$ & $\cos 2\theta$

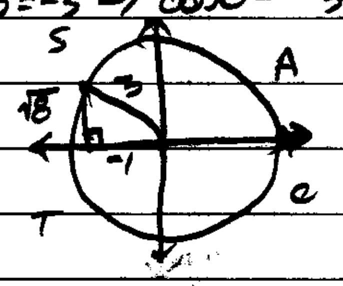
sol - $\sin 2\theta = 2\sin\theta\cos\theta$

$2\,\frac{2\sqrt{2}}{3}\cdot\left(-\frac{1}{3}\right) = -\frac{4\sqrt{2}}{9}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

$\sec\theta = -3 \Rightarrow \cos\theta = -\frac{1}{3}$



2. Half-angle Formula

$$\boxed{\begin{aligned} \sin\frac{u}{2} &= \pm\sqrt{\frac{1-\cos u}{2}} \\ \cos\frac{u}{2} &= \pm\sqrt{\frac{1+\cos u}{2}} \end{aligned}}$$

$\Rightarrow \sin^2 u = \frac{1-\cos^2 u}{2}$

$\Rightarrow \cos^2 u = \frac{1+\cos^2 u}{2}$

$\tan\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{1+\cos u}} \Rightarrow \tan^2 u = \frac{1-\cos^2 u}{1+\cos^2 u}$

EX Find the exact values of $\sin\frac{t}{2}$, $\cos\frac{t}{2}$ & $\tan\frac{t}{2}$ for $\cos t = \frac{4}{5}$ $0° < t < 90°$

First describe the quadrant that $\frac{t}{2}$ is in, $0° < t < 90° \Rightarrow 0° < \frac{t}{2} < 45°$ $\oplus$

$\sin\frac{t}{2} = \pm\sqrt{\frac{1-\cos t}{2}} = +\sqrt{\frac{1-\cos t}{2}} = \sqrt{\frac{1-4/5}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$

$\cos\frac{t}{2} = \pm\sqrt{\frac{1+\cos t}{2}} = +\sqrt{\frac{1+\cos t}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9/10}{2}} = \sqrt{\frac{9}{20}} = \frac{3\sqrt{5}}{10}$

$\tan\frac{t}{2} = \pm\sqrt{\frac{1-\cos t}{1+\cos t}} = +\sqrt{\frac{1-\cos t}{1+\cos t}} = \sqrt{\frac{1-4/5}{1+4/5}} = \sqrt{\frac{1/5}{9/5}} = \sqrt{1/9} = \frac{1}{3}$

Trig 7.4

EX Find the exact value of $\cos(67°30')$

Sol- $\cos(67°30') = \cos(\frac{u}{2})$

$67°30' = \frac{u}{2} \Rightarrow u = 2(67°30') = 67° \times 2 + 30' \times 2$

$= 134° + 60' = 135°$

$\cos(67°30') = \cos(\frac{135°}{2}) = \pm\sqrt{\frac{1+\cos u}{2}} = +\sqrt{\frac{1+\cos 135°}{2}} = \sqrt{\frac{1+(-\frac{\sqrt{2}}{2})}{2}}$

$= \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

EV Find exact value of $\sin 15°$ using half angle formula

$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

$= +\sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

Prove $4\sin\frac{x}{2}\cos\frac{x}{2} = 2\sin x$

$LS = 2 \cdot 2\sin\frac{x}{2}\cos\frac{x}{2}$	or $RS = 2\sin x$
$\frac{x}{2} = u$	$= 2\sin(2 \cdot \frac{x}{2})$
$2 \cdot 2\sin u\cos u$	$= 2(2\sin\frac{x}{2}\cos\frac{x}{2})$
$2 \cdot \sin 2(\frac{x}{2})$	$= 4\sin\frac{x}{2}\cos\frac{x}{2}$
$2\sin x$	$= LS$ ✓
$= RS$ ✓	