Trig 7.6 Inverse Functions

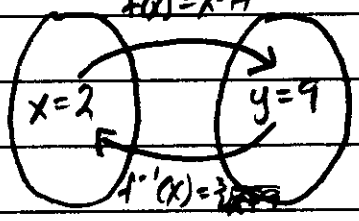
$f(x) = x^3 + 1$

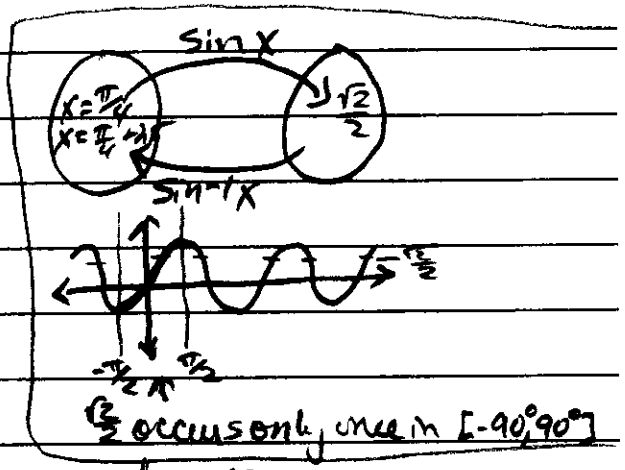
$y = x^3 + 1$

$x^3 = y - 1$

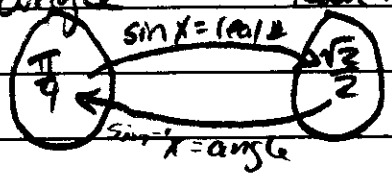
$x = \sqrt[3]{y-1}$

$f^{-1}(x) = \sqrt[3]{x-1}$





When we talk about inverse an function, we <u>always</u> talk about it in interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$\sin\theta$ = real #
$\sin^{-1}\theta$ = angle
$\cos\theta$ = real #
$\cos^{-1}\theta$ = angle
$\tan\theta$ = real #
$\tan^{-1}\theta$ = angle

$\tan^{-1}x = \arctan x$ $\sin^{-1}x = \arcsin x$ $\cos^{-1}x = \arccos x$

Given $\sin^{-1}x = a$ need to find $x = ?$ $a = 60°$

$\sin^{-1}x = 60°$

$\sin 60° = x$ $x = \frac{\sqrt{3}}{2}$

$\cos^{-1}y = \frac{4\pi}{3}$

$\cos\frac{4\pi}{3} = y$

$y = -\frac{1}{2}$

$\arctan x = -\frac{\pi}{4}$

$\tan -\frac{\pi}{4} = x$

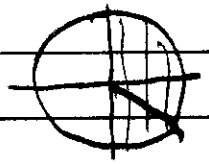
$x = -1$

$\sin^{-1}(-\frac{\sqrt{2}}{2}) = \text{angle} = \theta$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Ex. Let $\sin^{-1}(-\frac{\sqrt{2}}{2}) = \theta$, we know $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin\theta = -\frac{\sqrt{2}}{2}$

$\theta = -45°$



has to be 45°, not 315° since we can only use QI & QIV

Trig 7.6 cont

① Make sure same trig function & inverse function
② only look at left function
a) inverse function = angle
b) normal function = real value = obtain directly

(1) $\sin(\sin^{-1}(\frac{1}{2}))$ $\sin^{-1}(\sin\frac{5\pi}{6})$ $\sin^{-1}(\sin\frac{5\pi}{3})$ $\sin(\sin^{-1}\sqrt{2})$

$\{\frac{1}{2}$ no solution $\sin\theta = \sin\frac{5\pi}{6}$ $\sin^{-1}(\sin\frac{5\pi}{3}) = \theta$ $\{\sqrt{2}$ no solution

$\boxed{\frac{1}{2}}$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ $\sin\theta = \sin\frac{5\pi}{3}$ $\boxed{\text{no solution}}$

$\boxed{\theta = \frac{\pi}{6}}$ $\boxed{\theta = -\frac{\pi}{3}}$

(2.) $\cos(\cos^{-1}(\frac{1}{2}))$ $\cos^{-1}(\cos(\frac{5\pi}{6}))$ $\cos^{-1}(\cos\frac{5\pi}{3})$ $\cos(\cos^{-1}(\sqrt{2}))$

$\{\frac{1}{2}$ no solution $\cos^{-1}\cos\frac{5\pi}{6} = \theta$ $\cos^{-1}(\cos\frac{5\pi}{3}) = \theta$ $\{\sqrt{2}$ no solution

$\boxed{\frac{1}{2}}$ $\cos\theta = \cos\frac{5\pi}{6}$ $\cos\theta = \cos\frac{5\pi}{3}$ $\boxed{\text{no solution}}$

$\boxed{\theta = \frac{5\pi}{6}}$ $\boxed{\theta = -\frac{\pi}{3}}$

(3) $\tan(\tan^{-1}(\frac{1}{2}))$ $\tan^{-1}(\tan(\frac{5\pi}{6}))$ $\tan^{-1}(\tan(\frac{5\pi}{3}))$ $\tan(\tan^{-1}(\sqrt{2}))$

$\boxed{\frac{1}{2}}$ $\tan\theta = \tan\frac{5\pi}{6}$ $\tan\theta = \tan\frac{5\pi}{3}$ $\boxed{\sqrt{2}}$

$\boxed{\theta = -\frac{\pi}{6}}$ $\boxed{\theta = -\frac{\pi}{3}}$

On test $\sin(\sin^{-1}(\frac{\pi}{2}))$

$\{\frac{\pi}{2}$ no solution $\boxed{\text{no solution}}$ $\frac{\pi}{2} > 1$

★★★

$\sin^{-1}\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}\left[0, \pi\right]$

$\tan^{-1}\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Trig 7.6

$\arcsin \frac{\sqrt{3}}{2} = ?$

So Let $\theta = \arcsin \frac{\sqrt{3}}{2}$, we know $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60°$

$\sin^{-1}(4) = ?$

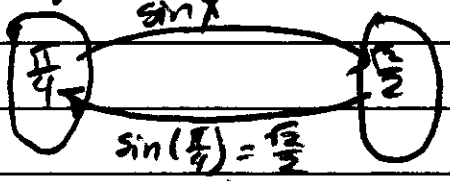
Let $\theta = \sin^{-1}(4)$, we know $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin\theta = 4$

$-1 \le \sin\theta \le 1$ no solution

$-1 \le \sin\theta \le 1$
$-1 \le \cos\theta \le 1$
$-\infty \le \tan\theta \le \infty$
$-\infty \le \cot \le \infty$

angle x real val y



must be in interval

$\sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$

cancel

$\sin^{-1}(\sin\frac{3\pi}{4}) = \frac{\pi}{4}$

$\neq \frac{3\pi}{4}$

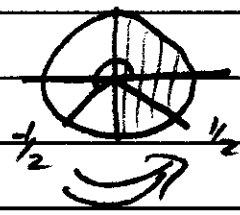
$\sin^{-1}(\sin\frac{3\pi}{4}) = \theta$ $\sin(\sin^{-1}x) =$ real value

$\sin\theta = \sin\frac{3\pi}{4}$

$\sin^{-1}(\sin(\frac{7\pi}{6})) = \theta$

$\sin\theta = \sin\frac{7\pi}{6}$

$\theta = -\frac{\pi}{6}$

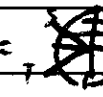


needs to be negative, $\sin\theta$ I or QIV

Trig 7.6 cont

1) $\cos(\arccos\frac{1}{2}) = \boxed{\frac{1}{2}}$

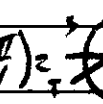
2) $\cos^{-1}(\cos\frac{5\pi}{6}) =$  $\boxed{\frac{5\pi}{6}}$

3) $\arccos(\cos\frac{5\pi}{4}) =$  $\boxed{\frac{3\pi}{4}}$

4) $\cos(\arccos\frac{\pi}{3}) =$ no solution

5) $\tan(\tan^{-1}100) =$ 100

6) $\tan^{-1}(\tan(-\frac{\pi}{6})) =$  $\boxed{-\frac{\pi}{6}}$

7) $\arctan(\tan\frac{5\pi}{4}) =$  $\boxed{\frac{\pi}{4}}$

8) $\tan^{-1}(\tan\frac{\pi}{6}) =$  $\boxed{\frac{\pi}{6}}$

$\sin^{-1}(\cos\frac{\pi}{2})$

Let $\theta = \sin^{-1}(\cos\frac{\pi}{2})$

$\sin\theta = \cos\frac{\pi}{2}$

$\sin\theta = 0$

$\theta = 0$

$\cos(\tan^{-1}(1))$

Let $\theta = \tan^{-1}(1)$

$\tan\theta = 1$

$\theta = 45°$

$\cos 45° = \boxed{\frac{\sqrt{2}}{2}}$