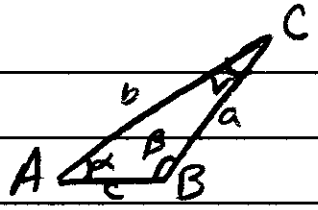
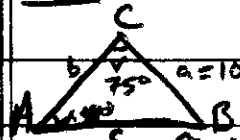
Trig 8.1 The Law of Sines



Law of Sines $\boxed{\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}}$

Ex 1 in $\triangle ABC$ $\alpha = 41°$ $\gamma = 75°$ $a = 10.5$, find the other angles & sides



$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} = \frac{\sin\beta}{b}$ $\beta = 180° - 75° - 41° = 64°$

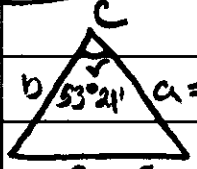
$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin 41°}{10.5} = \frac{\sin 64°}{b}$ $b \cdot \sin 41° = 10.5 \cdot \sin 64° \Rightarrow b = \frac{10.5 \sin 64°}{\sin 41°} = 14.4$

$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} = \frac{\sin 41°}{10.5} = \frac{\sin 75°}{c}$ $c \cdot \sin 41° = 10.5 \cdot \sin 75° \Rightarrow c = \frac{10.5 \sin 75°}{\sin 41°} = 15.5$

So $a = 10.5$ $b = 14.4$ $c = 15.5$

$\alpha = 41°$ $\beta = 64°$ $\gamma = 75°$

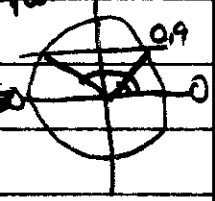
Ex 2 $\gamma = 53°21'$ $a = 140$ $c = 115$, find the other elements



$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$ $\frac{\sin\alpha}{140} = \frac{\sin 53°21'}{115}$ $115 \cdot \sin\alpha = 140 \cdot \sin 53°21'$

$\sin\alpha = \frac{140 \cdot \sin 53°21'}{115} = 0.9$ So $\alpha_1 = 77.55°$, $\alpha_2 = 180° - \alpha_1 = 102.05°$

Two Possible A



$\beta_1 = 180° - \alpha_1 - \gamma$
$= 180° - 77.55° - 53°20' = 45.11°$

$\beta = 180° - \alpha_2 - \gamma$
$= 180° - 102.05° - 53°20' = 24.21°$

$\frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$ $b_1 = \frac{c \cdot \sin\beta_1}{\sin\gamma} = \frac{115 \cdot \sin 45.11°}{\sin 53°20'} = 101.6$

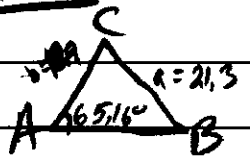
$b_2 = \frac{c \cdot \sin\beta_2}{\sin\gamma} = \frac{115 \cdot \sin 24.21°}{\sin 53°20'} = 58.8$

Thus we have 2 sets of solutions

Set 1	Set 2
$a = 140$ $b = 101.6$ $c = 115$	$a = 140$ $b = 58.8$ $c = 115$
$\alpha = 77.55°$ $\beta = 45.11°$ $\gamma = 53°21'$	$\alpha = 102.05°$ $\beta = 24.21°$ $\gamma = 53°21'$

Trig 8.1

Ex 3 $\alpha = 65.16°$ $a = 21.3$ $b = 18.9$, Find the other elements



$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$ $\frac{\sin 65.16}{21.3} = \frac{\sin\beta}{18.9}$ $18.9 \sin 65.16 = 21.3 \sin\beta$ $\sin\beta = \frac{18.9 \sin 65.16}{21.3} = 0.8$

$\beta_1 = 53.63°$

$\gamma_1 = 180° - \alpha - \beta_1$

$= 180° - 65.16° - 53.63°$

$= 61.20°$

$\beta_2 = 180° - 53.63° = 126.36°$

$\gamma_2 = 180° - \alpha - \beta_2$

$= 180° - 65.16° - 126.36°$

$= -11.97$

$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$ $\frac{\sin 65.16°}{21.3} = \frac{\sin 61.20°}{c}$ $c = \frac{21.3 \sin 61.20°}{\sin 65.16°} = 20.56$

Solution –

$a = 213$ $b = 18.9$ $c = 20.56$

$\alpha = 65.16°$ $\beta = 53.63°$ $\gamma = 61.20°$