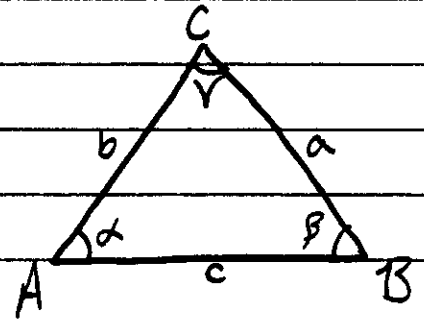
Trig 8.2 The Law of Cosines

Law of Cosines

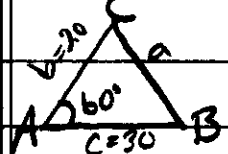
$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$



1st check if Law of Sines will solve the problem



$\alpha = 60°$ $b = 20$ $c = 30$ find the rest

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$ ← we can't use Law of Sines to solve

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$= 20^2 + 30^2 - 2\cdot 20\cdot 30\cdot\cos\alpha$ $(\cos 60° = 1/2)$

$= 400 + 900 - 2\cdot 20\cdot 30\cdot 1/2$

$= 1300 - 600$

$= 700$

$a = 10\sqrt{7}$

Find β using Law of Cosines

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$20^2 = (10\sqrt{7})^2 + 30^2 - 2\cdot 10\sqrt{7}\cdot 30\cdot\cos\beta$

$400 = 700 + 900 - 600\sqrt{7}\cos\beta$

$-1200 = -600\sqrt{7}\cos\beta$

$\frac{2}{\sqrt{7}} = \cos\beta$

$\beta = 41.7°$

Find β using Law of Sines

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \Rightarrow \frac{\sin 60°}{10\sqrt{7}} = \frac{\sin\beta}{20}$

$\sin\beta = \frac{20\sin 60°}{10\sqrt{7}} = 0.66$

Because $\beta \in (0, 180°)$, so β can be Q1, II

in Q1 $\beta_1 = 41.7°$

~~$\beta_2 = 180° - 41.7° = 138°$~~

Find γ

$\gamma = 180° - 60° - 41.7° = 78.3°$

Find γ

$\gamma_1 = 180° - 60° - 41.7° = 78.3°$

~~$\gamma_2 = 180° - 60° - 138.3° = -18.3°$ X~~

Trig 8.2 cont

Ex2 $\beta = 150^\circ$ $a = 150$ $c = 30$ find others $\quad \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

Using law of cosines

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$= 150^2 + 30^2 - 2(150)30 \cdot \cos\beta$

$= 22500 + 900 - 9000 \cdot \cos\beta$

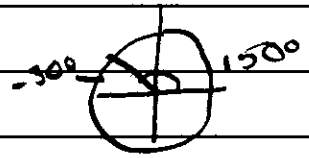
$= 23400 - 9000\cos\beta$

$= 23400 - 9000(-\cos 30)$

$= 23400 + 9000\frac{\sqrt{3}}{2}$

$= 23400 + 4500\sqrt{3}$

$b = 177$



Find α using law of cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$150^2 = 177^2 + 30^2 - 2 \cdot 177 \cdot 30\cos\alpha$

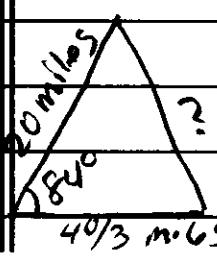
$2 \cdot 177 \cdot 30 \cdot \cos\alpha = 177^2 + 30^2 - 150^2$

$\cos\alpha = \frac{177^2 + 30^2 - 150^2}{2 \cdot 177 \cdot 30}$

$\alpha = 25^\circ$

$V = 180^\circ - 25^\circ - 150^\circ = 5^\circ$

Ex3 Two cars start from the same place, but drive in different directions. The angle between them is 84°. One car drives at 60mph, the other 40 mph, after 20 minutes, what is the distance between the cars?



$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$= 20^2 + \left(\frac{40}{3}\right)^2 - 2 \cdot 20 \cdot \frac{40}{3}\cos 84^\circ$

$a = 24$ miles