Trig 8.5 Trigonometric Form for Complex Numbers

Def A complex number is a number of the form $a+bi$, where a & b are real numbers and $i=\sqrt{-1}$, $i^2=-1$

examples: $3+2i$, $-7+\sqrt{20}\,i$

Absolute value of a complex number

$|a+bi| = \sqrt{a^2+b^2}$

$|3+2i| = \sqrt{3^2+2^2} = \sqrt{13}$

Ex. Find the absolute value $|3-4i|$

$|3+(-4)i| = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Ex Find the absolute value $|-25i|$

$|0-25i| = \sqrt{0^2+(-25)^2} = \sqrt{625} = 25$

Usually we denote a complex number as 'z', eg $z=3+i$

Let $z=a+bi$ $\quad |z| = \sqrt{a^2+b^2} = r$

$z = a+bi = (r\cos\theta) + (r\sin\theta)i = r\,cis\,\theta$

Ex $\sqrt{2}\,cis\left(\frac{\pi}{2}\right)$ so $r=\sqrt{2}$, $\theta=\frac{\pi}{2}$

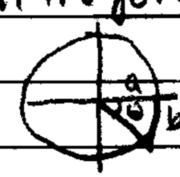
$\sqrt{2}\cos\left(\frac{\pi}{2}\right)+\sqrt{2}\sin\left(\frac{\pi}{2}\right)i$

$0+\sqrt{2}i$

$\sqrt{2}i$

Express the following complex number in the form $r\,cis\,\theta$

$z=1-i$ $\quad \theta\in[0,2\pi]$



$\tan\theta = \left|\frac{-1}{1}\right| = 1$

$\theta_r = 45°$

$a=1 \Rightarrow r=\sqrt{a^2+b^2}=\sqrt{1^2+(-1)^2}=\sqrt{2}$

$b=-1 \quad \theta = 315°$

$\sqrt{2}\,cis(315°)$

Trig 8.5 Complex Numbers

Two forms:

- $a+bi$ (algebra form) $\underline{a+bi}$
- Trig form $\underline{r\,\text{cis}\,\theta = r\cos\theta + ir\sin\theta}$

$a = r\sin\theta$
$b = r\cos\theta$

Given $z_1 = r_1 \text{cis}\,\theta_1 \quad z_2 = r_2 \text{cis}\,\theta_2$

then $z_1 \cdot z_2 = (r_1 r_2)\,\text{cis}(\theta_1+\theta_2)$

$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right)\text{cis}(\theta_1-\theta_2)$

$z_1 - z_2 \neq (r_1 - r_2)\,\text{cis}(\theta_1-\theta_2)$

$z_1 + z_2 \neq (r_1 + r_2)\,\text{cis}(\theta_1+\theta_2)$

EX1 $z_1 = \sqrt{2}+\sqrt{2}i \quad z_2 = 4 \quad$ Find $z_1 \cdot z_2 \quad \frac{z_1}{z_2}$

algebra: $z_1 + z_2 = \sqrt{2}+\sqrt{2}i \quad z_2 = 4+4\sqrt{3}i$

$z_1 + z_2 = (\sqrt{2}+\sqrt{2}i) + (4+4\sqrt{3}i)$

$= (\sqrt{2}+4) + (\sqrt{2}+4\sqrt{3})i$

$z_1 - z_2 = (\sqrt{2}+\sqrt{2}i) - (4+4\sqrt{3}i)$

$= \sqrt{2}-4 + (\sqrt{2}-4\sqrt{3})i$

$z_1 \cdot z_2 = (\sqrt{2}+\sqrt{2}i)(4-4\sqrt{3}i)$

$= 4\sqrt{2} + 4\sqrt{2}i - 4\sqrt{6}i - 4\sqrt{6}i^2$

$= (4\sqrt{2}+4\sqrt{6}) + (4\sqrt{2}-4\sqrt{6})i$

$\frac{z_1}{z_2} = \frac{\sqrt{2}+\sqrt{2}i}{4+4\sqrt{3}i} \cdot \frac{(4-4\sqrt{3}i)}{(4-4\sqrt{3}i)} = \frac{4\sqrt{2}-4\sqrt{6}i+4\sqrt{2}i-4\sqrt{6}i^2}{16+48} = \frac{4\sqrt{2} \ldots}{64}$

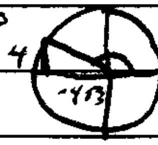
Trig 8.5

Express the following complex number in the form of $r \text{ cis } \theta$

$z = -4\sqrt{3} + 4i \qquad \theta \in [0, 2\pi]$

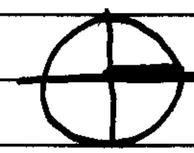
$a = -4\sqrt{3} \Rightarrow r = \sqrt{a^2+b^2} = \sqrt{(-4\sqrt{3})^2 + 4^2} = \sqrt{48+16} = \sqrt{64} = 8$

$b = 4 \qquad \theta = 150^\circ \qquad \tan\theta_r = \left|\frac{4}{-4\sqrt{3}}\right| = \frac{\sqrt{3}}{3} = 30^\circ$



$8 \text{ cis}\left(\frac{5\pi}{6}\right) \qquad \theta = 180^\circ - 30^\circ = 150^\circ$

EX $z = 32$ same as $z = 32 + 0i$



$a = 32 \Rightarrow r = \sqrt{a^2+b^2} = \sqrt{32^2 + 0^2} = 32$

$b = 0 \qquad \theta = 0^\circ \qquad 32 \text{ cis}(0)$

Suppose $z_1 = r_1 \text{ cis } \theta_1$, $z_2 = r_2 \text{ cis } \theta_2$

then $z_1 \cdot z_2 = r_1 r_2 \text{ cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{ cis}(\theta - \theta_2)$

$z_1^n = (r_1 \text{ cis } \theta_1)^2 = r_1^n \text{ cis}(n\theta)$

$(3+3i)^{100} = [3\sqrt{2} \text{ cis}(\frac{\pi}{4})]^{100} = 3\sqrt{2}^{100} \text{ cis}(\frac{\pi}{4} \cdot 100)$

$= (3\sqrt{2})^{100} \text{ cis } 25\pi$

$= (3\sqrt{2})^{100} \cos(25\pi) + (3\sqrt{2})^{100} \sin(25\pi) i$

$= (3\sqrt{2})^{100} \cos\pi + (3\sqrt{2})^{100} \sin\pi$

$= -(3\sqrt{2})^{100}$

Express $z_1 = \sqrt{2} \text{ cis}\left(\frac{3\pi}{4}\right)$ in the form $a+bi$

$r = \sqrt{2},\ \theta = \frac{3\pi}{4} \Rightarrow \sqrt{2}\cos\left(\frac{3\pi}{4}\right) + \sqrt{2}\sin\left(\frac{3\pi}{4}\right) i = 1 + 1i$

Express $z_2 = 1 + i$ in the form $r \text{ cis } \theta \qquad \tan\theta_r = \left|\frac{1}{1}\right| = 1$

$\theta_r = 45^\circ$

$a = 1 \Rightarrow r = \sqrt{1^2+1^2} = \sqrt{2}$

$b = 1 \qquad \theta = 45^\circ = \frac{\pi}{4} \qquad \sqrt{2} \text{ cis } \frac{\pi}{4}$