

1. (8 points) Convert radian to degree, or degree to radian

(a) $-\frac{\pi}{12}$

$$\frac{-\frac{\pi}{12}}{\pi} = \frac{\text{degree}}{180^\circ}$$

$$\text{degree} = \boxed{-15^\circ}$$

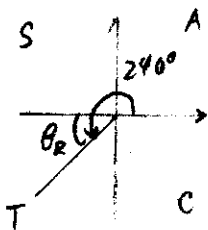
(b) 225°

$$\frac{225^\circ}{180^\circ} = \frac{\text{radian}}{\pi}$$

$$\text{radian} = \boxed{\frac{5\pi}{4}}$$

2. (5 points each) Using reference angles, give the exact value of the following:

a) $\cos(240^\circ) = -\cos 60^\circ = -\frac{1}{2}$

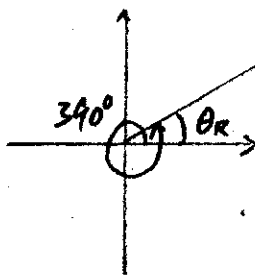


Let $\theta = 240^\circ$
 $\theta_R = 240^\circ - 180^\circ = \boxed{60^\circ}$
 $|\cos 240^\circ| = \cos 60^\circ$

In QIII, $\cos \theta < 0$

or, $\cos 240^\circ = -\cos 60^\circ = \boxed{-\frac{1}{2}}$

b) $\tan(390^\circ) =$

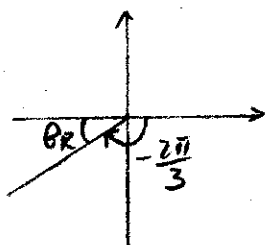


$\theta_R = 390^\circ - 360^\circ = \boxed{30^\circ}$

In QI, $\tan \theta > 0$

$\tan 390^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

c) $\csc\left(-\frac{2\pi}{3}\right) = \frac{1}{\sin\left(-\frac{2\pi}{3}\right)} = \frac{1}{-\sin\frac{\pi}{3}} = \frac{1}{-\frac{\sqrt{3}}{2}} = \boxed{-\frac{2}{\sqrt{3}}}$



$\theta_R = \pi - \frac{2\pi}{3} = \boxed{\frac{\pi}{3}}$

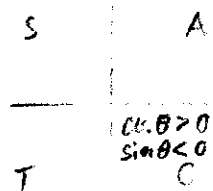
In QIII, $\sin \theta < 0$

3. (6 points) If $\sec \theta = (4)$ and $\sin \theta < 0$ find:

a) The quadrant in which θ is

Since $\sec \theta > 0$, then $\cos \theta > 0$

Since $\cos \theta > 0$ and $\sin \theta < 0$, θ in Q4.



b) $\cos \theta = \frac{1}{\sec \theta} = \frac{1}{4}$

c) $\sin \theta =$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{4^2 - 1^2} = \sqrt{15}$$

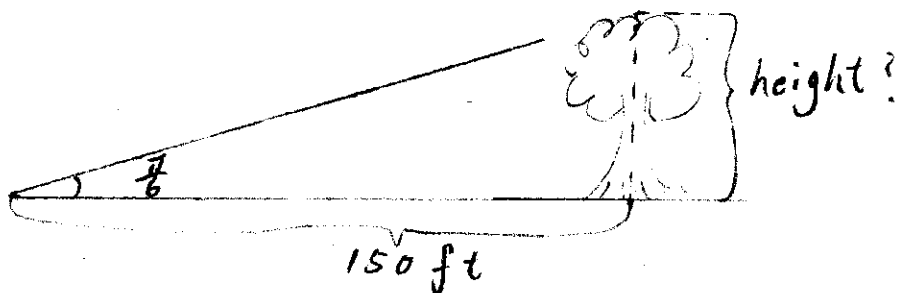
$$\sin \theta = \frac{y}{r} = \frac{\sqrt{15}}{-4}$$



$$\cos \theta = \frac{x}{r} = \frac{1}{4}$$

So, $x = 1, r = 4$

4. (4 points) When the top of a tree is viewed from a distance of 150 feet from the base, the angle of elevation is $\frac{\pi}{6}$. Find the height of the tree.



$$\tan \frac{\pi}{6} = \frac{\text{height}}{150}$$

$$\text{height} = 150 \cdot \tan \frac{\pi}{6}$$

$$\text{height} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\text{height} = \boxed{50\sqrt{3} \text{ ft}}$$

5. (12 points) Refer to the graph below.

a) Find the amplitude: $\boxed{3}$

b) Find the period: $\frac{11\pi}{4} - \frac{3\pi}{4} = \boxed{2\pi}$

c) Circle the correct equation for the graph.

$$y = -3\cos\left(\frac{1}{2}x\right)$$

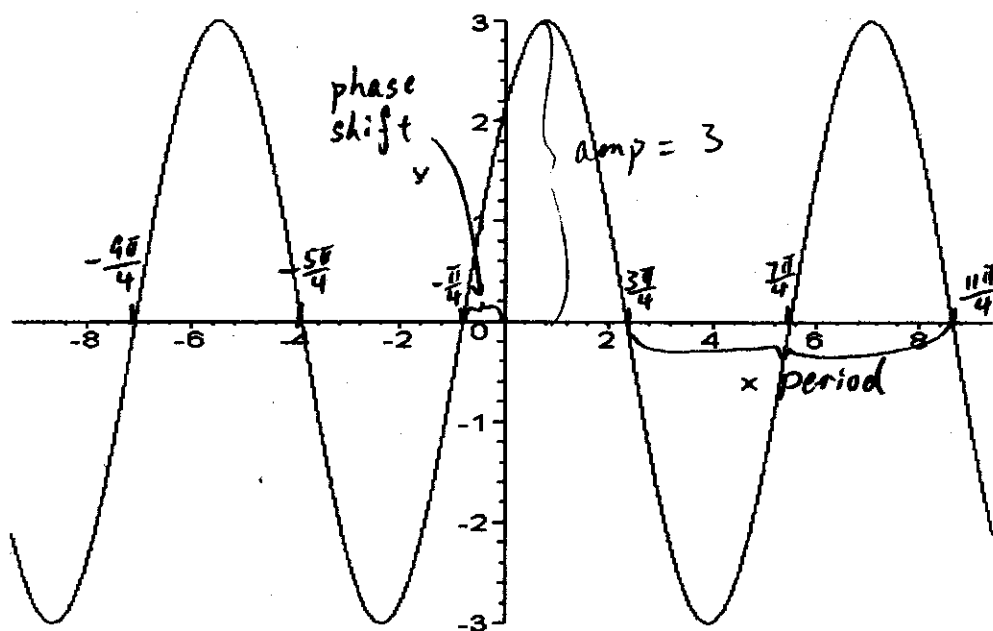
$$y = 3\cos\left(x + \frac{\pi}{4}\right)$$

$$y = 3\cos\left(\frac{1}{4}x + \frac{\pi}{4}\right)$$

$$y = -2\sin\left(\frac{1}{2}x\right)$$

$$y = 3\sin\left(x + \frac{\pi}{4}\right)$$

$$y = 3\sin\left(\frac{1}{4}x + \frac{\pi}{4}\right)$$



$$\text{Let } y = a \sin(bx + c)$$

$$1) a = 3,$$

$$2) \text{period} = \frac{2\pi}{|b|} = 2\pi \Rightarrow b = 1$$

$$3) \text{phase shift} = -\frac{c}{b} = -c = -\frac{\pi}{4} \Rightarrow c = \frac{\pi}{4}$$

6. (10 points) Verify the identity by transforming the left side of the equation into the right.

$$(\tan \theta + \cot \theta) \tan \theta = \sec^2 \theta$$

$$\begin{aligned} \text{L.H.S.} &= (\tan \theta + \cot \theta) \tan \theta \\ &= \left(\tan \theta + \frac{1}{\tan \theta} \right) \tan \theta \\ &= \left(\frac{\tan^2 \theta + 1}{\tan \theta} \right) \tan \theta \\ &= \tan^2 \theta + 1 \\ &= \sec^2 \theta \\ &= \text{R.H.S.} \end{aligned}$$

7. (5 points each)

- a) Find the length of the arc cut off by a sector with central angle 150° on a circle of diameter 16 cm.

$$\begin{aligned} S &= r \theta \quad \theta \text{ radian} \\ &= \frac{d}{2} \theta \\ &= \frac{16}{2} \cdot \frac{5\pi}{6} \\ S &= \boxed{\frac{20\pi}{3} \text{ cm}} \end{aligned} \quad \begin{aligned} \frac{150^\circ}{180^\circ} &= \frac{\theta \text{ radian}}{\pi} \\ \theta \text{ radian} &= \frac{5\pi}{6} \end{aligned}$$

- b) Find the area of the sector in part (a).

$$\begin{aligned} A &= \frac{1}{2} r^2 \theta \quad \theta \text{ radian} \\ &= \frac{1}{2} \left(\frac{16}{2} \right)^2 \cdot \frac{5\pi}{6} \\ &= \frac{1}{2} \cdot 64 \cdot \frac{5\pi}{6} \\ A &= \boxed{\frac{80\pi}{3} \text{ cm}^2} \end{aligned}$$

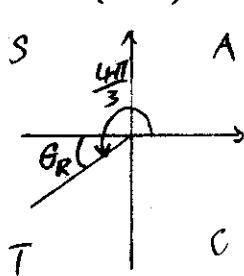
8. (10 points) Verify by transforming the left side into the right side:

$$\cot(-\theta)\cos(-\theta) + \sin(-\theta) = -\csc\theta$$

$$\begin{aligned} \text{L.H.S} &= \cot(-\theta)\cos(-\theta) + \sin(-\theta) \\ &= -\cot\theta \cdot \cos\theta - \sin\theta \\ &= -\frac{\cos\theta}{\sin\theta} \cdot \cos\theta - \sin\theta \cdot \frac{\sin\theta}{\sin\theta} \\ &= \frac{-\cos^2\theta - \sin^2\theta}{\sin\theta} \\ &= \frac{-(\cos^2\theta + \sin^2\theta)}{\sin\theta} = -\frac{1}{\sin\theta} = -\csc\theta = \text{R.H.S.} \end{aligned}$$

9. (5 points each) **Use a formula for negatives** to find the exact value of each of the following:

a) $\cot\left(-\frac{4\pi}{3}\right) = -\cot\frac{4\pi}{3}$

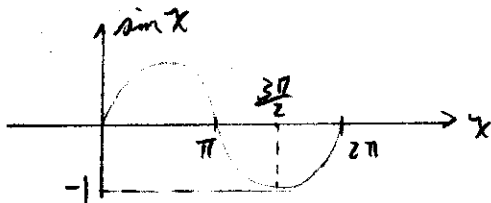


Let $\theta = \frac{4\pi}{3}$

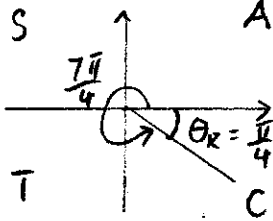
$$\theta_R = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

$$-\cot\frac{4\pi}{3} = -\left(+\cot\frac{\pi}{3}\right) = -\frac{1}{\tan\frac{\pi}{3}} = \boxed{-\frac{1}{\sqrt{3}}}$$

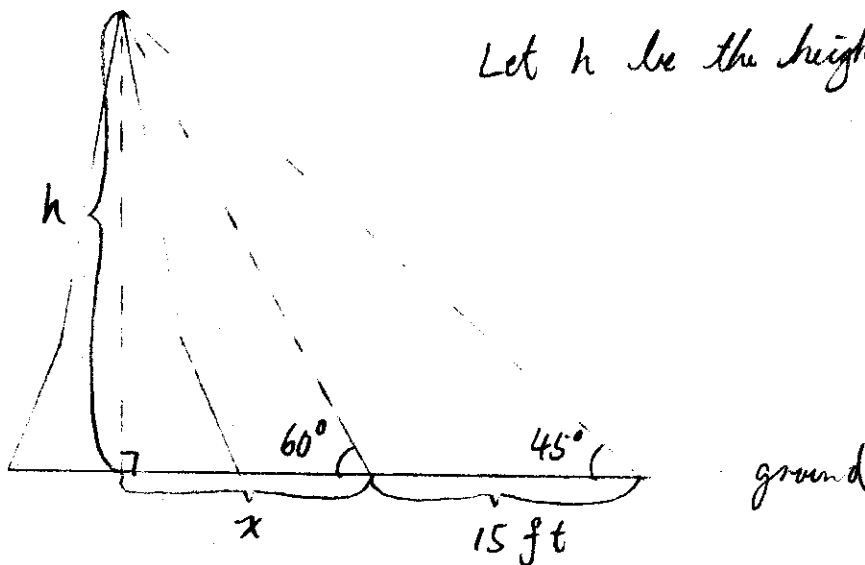
b) $\sin\left(-\frac{3\pi}{2}\right) = -\sin\frac{3\pi}{2} = -(-1) = \boxed{1}$



$$c) \sec\left(\frac{-7\pi}{4}\right) = \sec\frac{7\pi}{4} = \frac{1}{\cos\frac{7\pi}{4}} = \frac{1}{+\cos\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \boxed{\sqrt{2}}$$



10. (10 points) From a point on level ground the angle of elevation of the ground to the top of a tower is 45° . From a point 15 feet closer, the angle of elevation is 60° . Find the height of the tower.



Let h be the height of the tower.

$$\tan 45^\circ = \frac{h}{x+15} \quad \text{--- (1)} \qquad \tan 60^\circ = \frac{h}{x} \quad \text{--- (2)}$$

$$1 = \frac{h}{x+15}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = x + 15$$

$$x = \frac{h}{\sqrt{3}}$$

$$h = \frac{h}{\sqrt{3}} + 15$$

$$h - \frac{h}{\sqrt{3}} = 15$$

$$h\left(1 - \frac{1}{\sqrt{3}}\right) = 15$$

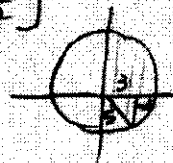
$$h = \boxed{15 / \left(1 - \frac{1}{\sqrt{3}}\right)} = \frac{15\sqrt{3}}{\sqrt{3} - 1} = \frac{15\sqrt{3}(\sqrt{3} + 1)}{2} = \frac{45 + 15\sqrt{3}}{2}$$

1. Evaluate: (6 points each)

a) $\cos\left[2\arcsin\left(-\frac{4}{5}\right)\right] =$

Let $\arcsin\left(-\frac{4}{5}\right) = \theta$ $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So $\sin\theta = -\frac{4}{5}$ θ in QIV



Thus $\cos\left[2\arcsin\left(-\frac{4}{5}\right)\right] = \cos 2\theta = 1 - 2\sin^2\theta = \boxed{-\frac{7}{25}}$

b) $\cot\left(\arcsin\left(\frac{12}{13}\right)\right) =$

Let $\theta = \arcsin\frac{12}{13}$ $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So $\sin\theta = \frac{12}{13}$ θ in QI.

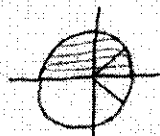
$\rightarrow \cos\theta = \frac{5}{13}$



Thus $\cot\left(\arcsin\frac{12}{13}\right) = \cot\theta = \frac{\cos\theta}{\sin\theta} = \boxed{\frac{5}{12}}$

c) $\cos^{-1}\left(\cos\left(\frac{11\pi}{6}\right)\right) = \alpha$ $\alpha \in [0, \pi]$ and $\cos\alpha = \cos\frac{11\pi}{6}$

So $\alpha = 2\pi - \frac{11\pi}{6} = \boxed{\frac{\pi}{6}}$



d) $\sin\left(\sin^{-1}\left(\frac{\pi}{2}\right)\right) =$ No solution since $\frac{\pi}{2} > 1$

2. For f , $f(x) = \frac{2}{7x-1}$

- a) Show that $f(x)$ is one to one. (5 points)
(Use the definition.)

$$f(a) = f(b) \Rightarrow \frac{2}{7a-1} = \frac{2}{7b-1} \Rightarrow a = b \Rightarrow 1-1$$

- b) Find $f^{-1}(x)$. (5 points)

$$y = \frac{2}{7x-1} \Rightarrow (7x-1)y = 2 \Rightarrow 7xy = y+2$$

$$\Rightarrow x = \frac{y+2}{7y}$$

$$f^{-1}(x) = \frac{y+2}{7y}$$

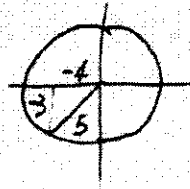
3. Express the following as a single sine or cosine and give the value of the expression:

(6 points)

$$\sin \frac{5\pi}{6} \cos \frac{\pi}{6} - \cos \frac{5\pi}{6} \sin \frac{\pi}{6} = \sin \left(\frac{5\pi}{6} - \frac{\pi}{6} \right) = \sin \frac{2\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

4. If θ is in Quadrant III and $\cot \theta = \frac{4}{3}$, find: (6 points)

$$\begin{aligned} \sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta \\ &= \frac{-4}{5} \end{aligned}$$



5. Find all the solutions of $2 \sin 2\theta - \sqrt{2} = 0$. (10 points)

Sol. $\sin 2\theta = \frac{\sqrt{2}}{2}$

$$2\theta = \frac{\pi}{4} + 2k\pi$$

or $2\theta = \left(\pi - \frac{\pi}{4}\right) + 2k\pi = \frac{3\pi}{4} + 2k\pi$

Sol. $\theta = \frac{\pi}{8} + k\pi$ or $\theta = \frac{3\pi}{8} + k\pi$



6. Verify the following: (7 points each)

a) $\frac{\sin^2(2\theta)}{\sin^2 \theta} = 4 - 4\sin^2 \theta$

$$\begin{aligned} L.S. &= \frac{(2 \sin \theta \cos \theta)^2}{\sin^2 \theta} = \frac{4 \cancel{\sin^2 \theta} \cos^2 \theta}{\cancel{\sin^2 \theta}} = 4 \cos^2 \theta = 4(1 - \sin^2 \theta) \\ &= 4 - 4\sin^2 \theta = R.S. \end{aligned}$$

$$b) \cos\left(x - \frac{5\pi}{2}\right) = \sin x$$

$$L S = \cos\left(x - \frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2} = 0 + \sin x = RS$$

7. Find the exact value of $\cos 15^\circ$. (10 points)
(Use $15^\circ = 45^\circ - 30^\circ$)

$$\begin{aligned} \cos(15^\circ) &= \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

8. Find the solutions of $2\cos^2 x + 3\cos x = -1$ in the interval $[0, 2\pi)$. (10 points)

$$2\cos^2 x + 3\cos x = -1$$

$$2\cos^2 x + 3\cos x + 1 = 0$$

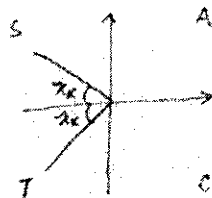
$$(2\cos x + 1)(\cos x + 1) = 0$$

$$2\cos x + 1 = 0 \quad \text{or} \quad \cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

$$\cos x = -1$$

$$x = \pi$$



$$\cos x_2 = |\cos x| = \frac{1}{2}$$

$$x_2 = \frac{\pi}{3}$$

$$\text{In Q II: } x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\text{In Q III: } x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

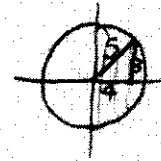
9. Find $\cos\left(\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(-\frac{4}{3}\right)\right)$

(10 points)

Sol. ① Let $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$ $\beta = \tan^{-1}\left(-\frac{4}{3}\right)$

② Then $\sin \alpha = \frac{3}{5}$ & α in QI

$\Rightarrow \cos \alpha = \frac{4}{5}$



and $\tan \beta = -\frac{4}{3}$ & β in QIV

$\Rightarrow \sin \beta = -\frac{4}{5}$, $\cos \beta = \frac{3}{5}$



Thus $\cos\left(\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(-\frac{4}{3}\right)\right)$

$= \cos(\alpha - \beta)$


$= \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$= \frac{4}{5} \cdot \frac{3}{5} + \frac{3}{5} \left(-\frac{4}{5}\right) = \boxed{0}$

1. (10 points)

a) Convert the point $(-2\sqrt{2}, 2\sqrt{2})$ from rectangular coordinates to polar coordinates with $r > 0$ and $0 \leq \theta < 2\pi$

$$\begin{cases} x = -2\sqrt{2} \\ y = 2\sqrt{2} \end{cases} \Rightarrow \begin{cases} r = \sqrt{x^2 + y^2} = 4 \\ \theta \end{cases}$$



$$|\tan \theta| = 1 \Rightarrow \theta = 45^\circ$$

$$\Rightarrow \theta = 135^\circ$$

$$S. (-2\sqrt{2}, 2\sqrt{2}) = \left(4, \frac{3\pi}{4}\right)$$

b) Convert the point $\left(5, \frac{7\pi}{6}\right)$ from polar to rectangular coordinates.

$$\begin{cases} r = 5 \\ \theta = \frac{7\pi}{6} \end{cases} \Rightarrow \begin{cases} x = r \cos \theta = 5 \cos \frac{7\pi}{6} = 5 \left(-\cos \frac{\pi}{6}\right) = -\frac{5\sqrt{3}}{2} \\ y = r \sin \theta = 5 \sin \frac{7\pi}{6} = 5 \left(-\sin \frac{\pi}{6}\right) = -\frac{5}{2} \end{cases}$$

$$S. \left(5, \frac{7\pi}{6}\right) = \left(-\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$$

2. (5 points) Express in the form $a + bi$: $-5 \operatorname{cis} \frac{5\pi}{2}$.

$$\begin{cases} r = -5 \\ \theta = \frac{5\pi}{2} \end{cases} \Rightarrow \begin{cases} a = r \cos \theta = (-5) \cos \frac{5\pi}{2} = 0 \\ b = r \sin \theta = (-5) \sin \frac{5\pi}{2} = -5 \end{cases}$$

$$S. -5 \operatorname{cis} \frac{5\pi}{2} = -5i$$

3. (10 points) Find the value of the following:

a) $|5 - 8i| = \sqrt{5^2 + (-8)^2} = \sqrt{25 + 64} = \sqrt{89}$

b) $|\sqrt{3}i| = \sqrt{3}$

4. (10 points) Use DeMoivre's Theorem to change $\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^5$ to $a + bi$ form.

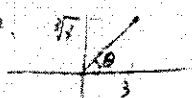
$$-\frac{\sqrt{3}}{2} - \frac{1}{2}i = 1 \cdot \text{cis}\left(\frac{7\pi}{6}\right)$$

$$\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^5 = \text{cis}\left(\frac{7\pi}{6} \times 5\right) = \text{cis}\left(\frac{35\pi}{6}\right) = \text{cis}\left(-\frac{11\pi}{6}\right)$$

$$= 1 \cdot \text{cis}\frac{11\pi}{6} + i \cdot \text{sin}\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - i \cdot \frac{1}{2}$$

5. (5 points) Express $3 + 3\sqrt{3}i$ in trigonometric form with $0 \leq \theta < 2\pi$.

$$\begin{cases} a = 3 \\ b = 3\sqrt{3} \end{cases} \Rightarrow \begin{cases} r = \sqrt{a^2 + b^2} = 6 \\ \theta = \arctan\left(\frac{b}{a}\right) = \arctan(\sqrt{3}) \Rightarrow \theta = \frac{\pi}{3} \end{cases}$$



So $3 + 3\sqrt{3}i = 6 \text{cis}\left(\frac{\pi}{3}\right)$

6. (12 points) Given $z_1 = 4i$ and $z_2 = 3 + 3i$.

- a) Find $z_1 z_2$ and put your answer in $a + bi$ form.

$$z_1 z_2 = 4i(3 + 3i) = 12i - 12 = -12 + 12i$$

or $z_1 = 4i = 4 \text{cis}\frac{\pi}{2}$

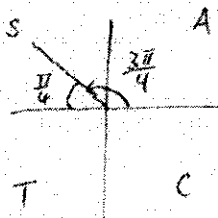
$$z_2 = 3 + 3i = 3\sqrt{2} \text{cis}\frac{\pi}{4}$$

$$z_1 \cdot z_2 = 4 \cdot 3\sqrt{2} \text{cis}\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$$

$$= 12\sqrt{2} \left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)$$

$$= 12\sqrt{2} \cdot \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}\right)$$

$$= \boxed{-12 + 12i}$$



b) Find $\frac{z_1}{z_2}$ and put your answer in a + bi form.

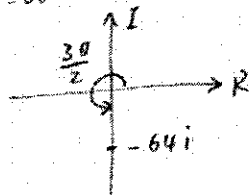
$$\frac{z_1}{z_2} = \frac{4i}{3+i} = \frac{4i(3-i)}{9+9} = \frac{12i+12}{18} = \frac{2}{3}(1+i)$$

$$\begin{aligned} \text{or } \frac{z_1}{z_2} &= \frac{4}{3\sqrt{2}} \operatorname{cis}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{4}{3\sqrt{2}} \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \\ &= \frac{4}{3\sqrt{2}} \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) \\ &= \boxed{\frac{2}{3} + \frac{2}{3}i} \end{aligned}$$

7. (8 points) Find the two square roots of $-64i$. Put your answer in a + bi form.

$$w_k = \sqrt[n]{r} \left[\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right) \right] \text{ where } k=0,1,2,\dots,n-1$$

$$\text{let } z = -64i = 64 \operatorname{cis}\frac{3\pi}{2}, \quad n=2$$



$$(z)^{\frac{1}{2}} = w_k = \sqrt{64} \operatorname{cis}\left(\frac{\frac{3\pi}{2} + 2k\pi}{2}\right), \quad k=0,1$$

$$w_0 = 8 \operatorname{cis}\frac{3\pi}{4} = 8\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 8\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$w_0 = \boxed{-4\sqrt{2} + i4\sqrt{2}}$$

$$w_1 = 8 \operatorname{cis}\left(\frac{7\pi}{4}\right) = 8\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= 8\left(\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= 4\sqrt{2} - i4\sqrt{2}$$

8. (10 points) Find the four fourth roots of $-5\sqrt{2} + 5\sqrt{2}i$. Leave your answers in trigonometric form.

$$z = -5\sqrt{2} + 5\sqrt{2}i = 10 \operatorname{cis}\left(\frac{3\pi}{4}\right)$$

$$\sqrt[4]{z} = \left[10 \operatorname{cis}\left(\frac{3\pi}{4}\right)\right]^{\frac{1}{4}} = \sqrt[4]{10} \operatorname{cis}\left(\frac{\frac{3\pi}{4} + 2k\pi}{4}\right) \quad k=0,1,2,3$$

$$w_0 = \sqrt[4]{10} \operatorname{cis}\frac{3\pi}{16}$$

$$w_4 = \sqrt[4]{10} \operatorname{cis}\left(\frac{27\pi}{16}\right)$$

$$w_1 = \sqrt[4]{10} \operatorname{cis}\left(\frac{11\pi}{16}\right)$$

$$w_2 = \sqrt[4]{10} \operatorname{cis}\left(\frac{19\pi}{16}\right)$$

9. (10 points) Prove by induction $1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$ is true for all the Natural Numbers. (Show ALL YOUR WORK, each step must be shown.)

Proof

① $n=1$

$$3n-2 = 1 \Rightarrow LS = 1$$

$$RS = \frac{1 \cdot (3 \cdot 1 - 1)}{2} = 1$$

$$LS = RS$$

② Assume

$$n=k$$

$$1+4+7+\dots+(3k-2) = \frac{k(3k-1)}{2} \quad \text{⊗}$$

③ then

$$n=k+1$$

$$3n-1 = 3(k+1)-2$$

$$LS = \underbrace{1+4+7+\dots+(3k-2)}_{\text{⊗}} + \boxed{3(k+1)-2}$$

$$= \frac{k(3k-1)}{2} + \boxed{3(k+1)-2}$$

$$= \frac{3k^2 - k + 6k + 2}{2} = \frac{3k^2 + 5k + 2}{2}$$

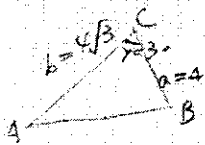
$$RS = \frac{(k+1)(3(k+1)-1)}{2} = \frac{(k+1)(3k+2)}{2}$$

$$= \frac{3k^2 + 5k + 2}{2}$$

$$\therefore LS = RS$$

④ Thus $1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2} \quad \forall n \geq 1$

10. (10 points) In triangle ABC, $\gamma = 30^\circ$, $b = 4\sqrt{3}$, $a = 4$. Find the possible values of β, α, c .



So $C^2 = a^2 + b^2 - 2ab \cos \gamma$

$$= 16 + 48 - 2 \times 4 \times 4\sqrt{3} \cos 30^\circ$$

$$= 64 - \frac{16}{2} \sqrt{3} \times \frac{\sqrt{3}}{2} = 64 - 48 = 16$$

$$C = 4$$

② Find α

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{48 + 16 - 16}{2 \times 4\sqrt{3} \times 4} = \frac{\sqrt{3}}{2}$$

$$\alpha = \frac{\pi}{6} = 30^\circ$$

③ Find β

$$\beta = \pi - \alpha - \gamma = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120^\circ$$