

Trig Exam #2 Review

PS 1

1-1 function $f^{-1}(x)$

① Show $f(x) = \frac{3}{4x-12}$ is a 1-1 function

② Find $f^{-1}(x)$

① Let $f(a) = f(b)$

$$\frac{3}{4a-12} = \frac{3}{4b-12}$$

$$\cancel{3}(4b-12) = \cancel{3}(4a-12)$$

$$4b-12 = 4a-12$$

$$4b = 4a$$

$$b = a \quad \checkmark \text{ 1-to-1 function}$$

② $f^{-1}(x) = \frac{3}{4x-12}$

$$y = \frac{3}{4x-12}$$

$$y(4x-12) = 3$$

$$4xy - 12y = 3$$

$$4xy = 3 + 12y$$

$$x = \frac{3 + 12y}{4y}$$

$$f^{-1}(x) = \frac{3 + 12x}{4x}$$

Express the following as one single sine or cosine, and give the value of the expression.

$$\sin \frac{\pi}{2} \cos \frac{3\pi}{3} - \cos \frac{\pi}{2} \sin \frac{3\pi}{3} \Rightarrow \sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\sin\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$$

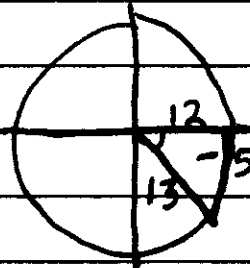
$$\sin\left(\frac{3\pi - 4\pi}{6}\right) = \sin -\frac{\pi}{6} = -\sin \frac{\pi}{6} = \boxed{-\frac{1}{2}}$$

Let β in (QIV) and $\cot \beta = -\frac{12}{5}$

Find $\sin\left(\frac{\pi}{2} - \beta\right)$

$$\sin\left(\frac{\pi}{2} - \beta\right) = \cos \beta$$

$$\cos \beta = \boxed{\frac{12}{13}}$$



Find all solutions of $2\cos 2\beta - \sqrt{2} = 0$

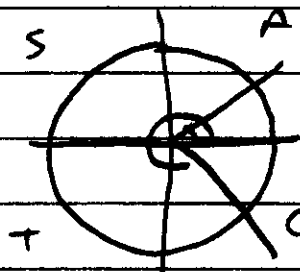
sol ① Find solution in $[0, 2\pi)$

$$2\cos 2\beta - \sqrt{2} = 0$$

$$\cos 2\beta = \frac{\sqrt{2}}{2} \text{ (positive so } I \text{ or } IV)$$

$$QI \ 2\beta = \frac{\pi}{4} \Rightarrow \beta = \frac{\pi}{8}$$

$$QIV \ 2\beta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \Rightarrow \beta = \frac{7\pi}{8}$$



sol ② all solutions $\beta = \frac{\pi}{8} + 2k\pi$ or $\frac{7\pi}{8} + 2k\pi$

Verify $\sin(x - \frac{5\pi}{2}) = -\cos x$

$$LS = \sin(x - \frac{5\pi}{2})$$

$$= \sin(x - (2\pi + \frac{\pi}{2}))$$

$$= \sin(x - 2\pi - \frac{\pi}{2})$$

$$= \sin(x - \frac{\pi}{2})$$

$$= \sin(-(x - \frac{\pi}{2}))$$

$$= -\sin(\frac{\pi}{2} - x)$$

$$= -\cos x$$

$$RS = -\cos x$$

$$LS = RS \checkmark$$

Find exact value of $\sin 15^\circ$ (use $15^\circ = 45^\circ - 30^\circ$)

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Trig Exam #2 Review

pg 3

Find the solutions of $2\sin^2 x + 3\sin x = -1$

$$2\sin^2 x + 3\sin x = -1$$

$$2\sin^2 x + 3\sin x + 1 = 0 \quad (\text{let } y = \sin x)$$

$$2y^2 + 3y + 1 = 0$$

$$(2y+1)(y+1) = 0$$

$$2y+1=0 \quad y+1=0$$

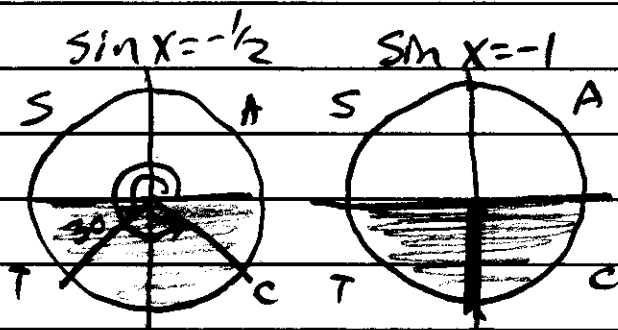
$$y = -\frac{1}{2} \quad y = -1$$

$$\sin x = -\frac{1}{2} \quad \sin x = -1$$

$$\text{in QIII } x = 180^\circ + 30^\circ = \boxed{210^\circ}$$

$$\text{in QIV } x = 360^\circ - 30^\circ = \boxed{330^\circ}$$

$$\text{in QIII, QIV } x = \boxed{270^\circ}$$



Verify $\frac{\sin^2(2\theta)}{\sin^2\theta} = 4 - 4\sin^2\theta$

$$LS = \frac{\sin^2(2\theta)}{\sin^2\theta}$$

$$= \frac{(\sin(2\theta))^2}{\sin^2\theta}$$

$$= \frac{(2\sin\theta\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{4\sin^2\theta\cos^2\theta}{\sin^2\theta}$$

$$= 4\cos^2\theta$$

$$= 4(1 - \sin^2\theta)$$

$$= 4 - 4\sin^2\theta$$

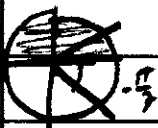
$$RS = 4 - 4\sin^2\theta$$

$$LS = RS \checkmark$$

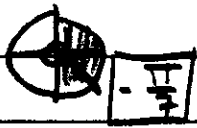
TRIG REVIEW EXAM #2

pg 4

$$\cos^{-1}(\cos \frac{13\pi}{7}) \quad \sin^{-1}(\sin \frac{13\pi}{7}) \quad \sin(\sin^{-1}(\frac{1}{2})) \quad \cos(\cos^{-1}(\sqrt{2}))$$



$$\boxed{-\frac{1}{7}}$$



$$\boxed{-\frac{1}{7}}$$

no solution

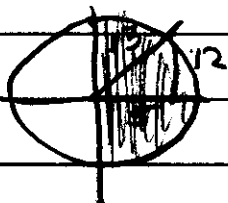
no solution

$$\tan(\sin^{-1}(\frac{12}{13}))$$

let $\theta = \sin^{-1}(\frac{12}{13})$

$$\sin \theta = \frac{12}{13}$$

$$\boxed{\tan \theta = \frac{12}{5}}$$



$$\cot(\cos^{-1}(\frac{12}{13}))$$

let $\theta = \cos^{-1}(\frac{12}{13})$

$$\cos \theta = \frac{12}{13}$$

$$\boxed{\cot \theta = \frac{12}{5}}$$



$$\sin(2 \arcsin(-\frac{3}{5}))$$

let $\theta = \arcsin(-\frac{3}{5})$

$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$2 \sin \theta \cos \theta$$

$$2(-\frac{3}{5})(\frac{4}{5}) = \boxed{-\frac{24}{25}}$$



$$\cos(2 \arcsin(-\frac{3}{5}))$$

let $\theta = \arcsin(-\frac{3}{5})$

$$\sin \theta = -\frac{3}{5}$$

$$1 - 2 \sin^2 \theta$$

$$1 - 2(-\frac{3}{5})^2$$

$$1 - \frac{18}{25}$$

$$\boxed{\frac{7}{25}}$$

