Trig Exam #2 Review

1-1 function $f^{-1}(x)$

① Show $f(x)=\frac{3}{4x-12}$ is a 1-1 function

② Find $f^{-1}(x)$

① Let $f(a)=f(b)$

$\frac{3}{4a-12}=\frac{3}{4b-12}$

$\cancel{3}(4b-12)=\cancel{3}(4a-12)$

$4b-\cancel{12}=4a-\cancel{12}$

$\cancel{4}b=\cancel{4}a$

$\boxed{b=a \checkmark \text{ 1 to 1 function}}$

② $f^{-1}(x)=\frac{3}{4x-12}$

$y=\frac{3}{4x-12}$

$y(4x-12)=3$

$4xy-12y=3$

$4xy=3+12y$

$x=\frac{3+12y}{4y}$

$\boxed{f^{-1}(x)=\frac{3+12x}{4x}}$

Express the following as one single sine or cosine, and give the value of the expression.

$\sin\frac{\pi}{2}\cos\frac{2\pi}{3}-\cos\frac{\pi}{2}\sin\frac{2\pi}{3} \Rightarrow \sin(u-v)=\sin u\cos v-\cos u\sin v$

$\sin\left(\frac{\pi}{2}-\frac{2\pi}{3}\right)$

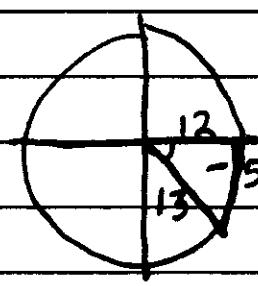
$\sin\left(\frac{3\pi-4\pi}{6}\right)=\sin-\frac{\pi}{6}=-\sin\frac{\pi}{6}=\boxed{-\frac{1}{2}}$

Let β in QIV and $\cot\beta=-\frac{12}{5}$

Find $\sin\left(\frac{\pi}{2}-\beta\right)$

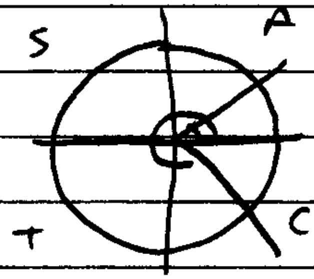
$\sin\left(\frac{\pi}{2}-\beta\right)=\cos\beta$

$\cos\beta=\boxed{\frac{12}{13}}$



Find all solutions of $2\cos 2\beta - \sqrt{2} = 0$

sol ① Find solution in $[0, 2\pi]$



$2\cos 2\beta - \sqrt{2} = 0$

$\cos 2\beta = \frac{\sqrt{2}}{2}$ (positive so QI & QIV)

QI $2\beta = \frac{\pi}{4} \Rightarrow \beta = \frac{\pi}{8}$

QIV $2\beta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \Rightarrow \beta = \frac{7\pi}{8}$

sol ② all solutions $\beta = \boxed{\frac{\pi}{8} + 2k\pi \text{ or } \frac{7\pi}{8} + 2k\pi}$

Verify $\sin(x - \frac{5\pi}{2}) = -\cos x$

$LS = \sin(x - \frac{5\pi}{2})$

$= \sin(x - (2\pi + \frac{\pi}{2}))$

$= \sin(x - 2\pi - \frac{\pi}{2}$

$= \sin(x - \frac{\pi}{2})$

$= \sin(-(x - \frac{\pi}{2}))$

$= -\sin(\frac{\pi}{2} - x)$

$= -\cos x$

$RS = -\cos x$

$LS = RS$ ✓

Find exact value of $\sin 15°$ (use $15° = 45° - 30°$)

$\sin 15° = \sin(45° - 30°$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$

Find the solutions of $2\sin^2 x + 3\sin x = -1$

$2\sin^2 x + 3\sin x = -1$

$2\sin^2 x + 3\sin x + 1 = 0$ (Let $y = \sin x$)

$2y^2 + 3y + 1 = 0$

$(2y+1)(y+1) = 0$

$2y+1=0 \quad y+1=0$

$y = -\frac{1}{2} \quad y = -1$

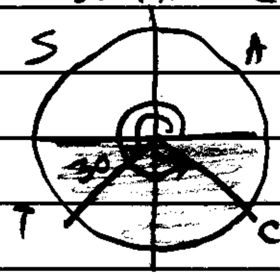
$\sin x = -\frac{1}{2} \quad \sin x = -1$

in QIII $x = 180° + 30° = \boxed{210°}$

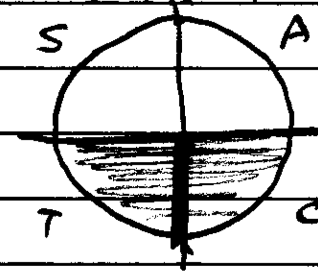
in QIV $x = 360° - 30° = \boxed{330°}$

in QIII, QIV $x = \boxed{270°}$

$\sin x = -\frac{1}{2}$



$\sin x = -1$



Verify $\frac{\sin^2(2\theta)}{\sin^2\theta} = 4 - 4\sin^2\theta$

$LS = \frac{\sin^2\theta(2\theta)}{\sin^2\theta}$

$= \frac{(\sin(2\theta))^2}{\sin^2\theta}$

$= \frac{(2\sin\theta\cos\theta)^2}{\sin^2\theta}$

$= \frac{4\sin^2\theta\cos^2\theta}{\sin^2\theta}$

$= 4\cos^2\theta$

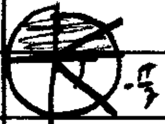
$= 4(1 - \sin^2\theta)$

$= 4 - 4\sin^2\theta$

$RS = 4 - 4\sin^2\theta$ $LS = RS$ ✓

TRIG REVIEW EXAM #2

$\cos^{-1}\left(\cos \frac{13\pi}{7}\right)$



$\boxed{\frac{\pi}{7}}$

$\sin^{-1}\left(\sin \frac{13\pi}{7}\right)$

$\boxed{-\frac{\pi}{7}}$

$\sin\left(\sin^{-1}\left(\frac{\pi}{2}\right)\right)$

no solution

$\cos\left(\cos^{-1}(\sqrt{2})\right)$

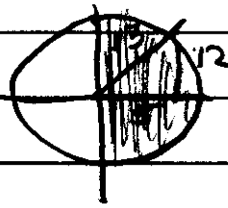
no solution

$\tan(\underbrace{\sin^{-1}\left(\tfrac{12}{13}\right)}_{\theta})$

Let $\theta = \sin^{-1}\left(\frac{12}{13}\right)$

$\sin\theta = \frac{12}{13}$

$\boxed{\tan\theta = \frac{12}{5}}$

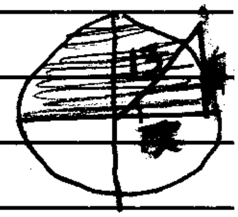


$\cot(\underbrace{\cos^{-1}\left(\tfrac{12}{13}\right)}_{\theta})$

Let $\cos^{-1}\left(\frac{12}{13}\right) = \theta$

$\cos\theta = \frac{12}{13}$

$\boxed{\cot\theta = \frac{12}{5}}$



$\sin(2\,\underbrace{\arcsin\left(-\tfrac{3}{5}\right)}_{\theta})$

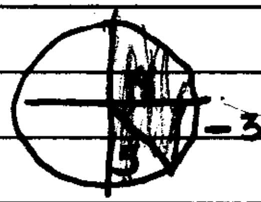
Let $\theta = \arcsin\left(-\frac{3}{5}\right)$

$\sin\theta = -3/5$

$\cos\theta = 4/5$

$2\sin\theta\cos\theta$

$2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$



$\cos(2\,\underbrace{\arcsin\left(-3/5\right)}_{\theta})$

Let $\theta = \arcsin\left(-3/5\right)$ $\quad \cos(2\theta) = 1 - 2\sin^2\theta$

$\sin\theta = -3/5$

$1 - 2\sin^2\theta$

$1 - 2\left(-\frac{3}{5}\right)^2$

$1 - \frac{18}{25}$

$\boxed{\frac{7}{25}}$