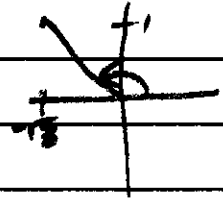


Trig Exam 3 Review

Conversion between $(x, y) \leftrightarrow (r, \theta)$

1.a. $(-\sqrt{3}, 1) \rightarrow (r, \theta)$

$$\begin{cases} a = -\sqrt{3} & r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2 \\ b = 1 & \tan \theta_r = \left| \frac{1}{-\sqrt{3}} \right| = \frac{\sqrt{3}}{3} \quad \theta_r = \frac{\pi}{6} \\ & \theta = \frac{5\pi}{6} \end{cases}$$



$(r, \theta) = (2, \frac{5\pi}{6})$

1.b. $(7, \frac{11\pi}{6}) \rightarrow (x, y)$

$$\begin{aligned} r = 7 & \Rightarrow x = r \cos \frac{11\pi}{6} = 7(\pm \cos \theta_r) = 7(\cos \frac{\pi}{6}) = 7 \cdot \frac{\sqrt{3}}{2} = \frac{7\sqrt{3}}{2} \\ \theta = \frac{11\pi}{6} & \Rightarrow y = r \sin \frac{11\pi}{6} = 7(\pm \sin \theta_r) = 7(-\sin \frac{\pi}{6}) = 7 \cdot \frac{1}{2} = \frac{7}{2} \end{aligned}$$

2. Compute $|7 - 4i| = \sqrt{7^2 + 4^2} = \sqrt{65}$

$|-\sqrt{3}i| = \sqrt{0^2 + (\sqrt{3})^2} = \sqrt{3}$

$|-\sqrt{3}i| = \sqrt{(\sqrt{3}i)^2} = \sqrt{3}$

3. Express $10 \cos(\frac{5\pi}{6})$ as $a + bi$ form

$$\begin{cases} r = 10 \\ \theta = \frac{5\pi}{6} \end{cases} \Rightarrow \begin{cases} x = r \cos \frac{5\pi}{6} = 10(\pm \cos \frac{\pi}{6}) = -5\sqrt{3} \\ y = r \sin \frac{5\pi}{6} = 10(\pm \sin \frac{\pi}{6}) = 5i \end{cases} \quad (-5\sqrt{3} + 5i)$$

4. Use DeMoivre's Thm to change $(-\frac{1}{2} - \frac{\sqrt{3}}{2}i)^6$ to $a + bi$ form

$$\begin{cases} a = -\frac{1}{2} \\ b = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow r = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\tan \theta_r = \left| \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} \right| = \frac{\sqrt{3}}{1} = \sqrt{3} \quad \theta_r = \frac{\pi}{3} \quad \theta = \frac{4\pi}{3}$$

$= (1 \operatorname{cis}(\frac{4\pi}{3}))^6 \quad (z^n = r^n \operatorname{cis}(n \cdot \theta))$

$= 1^6 \operatorname{cis}(6 \cdot \frac{4\pi}{3})$

$= \operatorname{cis}(8\pi)$

$= \cos 8\pi + i \sin 8\pi$

$= 1 + i \cdot 0 = 1$



Trig Exam 3 Review

⑤ $z_1 = 5i$ $z_2 = \sqrt{2} + \sqrt{2}i$

Find $z_1 \cdot z_2$

Find $\frac{z_1}{z_2}$

$$\begin{aligned} z_1 \cdot z_2 &= 5i(\sqrt{2} + \sqrt{2}i) \\ &= 5\sqrt{2}i + 5\sqrt{2}i^2 \\ &= -5\sqrt{2} + 5\sqrt{2}i \end{aligned}$$

$$\frac{5i}{\sqrt{2} + \sqrt{2}i} \cdot \frac{(\sqrt{2} - \sqrt{2}i)}{(\sqrt{2} - \sqrt{2}i)} = \frac{5\sqrt{2}i + 5\sqrt{2}}{2 + 2}$$

$$= \frac{5\sqrt{2}}{4} + \frac{5\sqrt{2}}{4}i$$

2 problems of this type

⑥ Find the cubic roots of $-27i$, Put answer in $a+bi$ form

$$z = -27i$$

$$w_1 = \sqrt[3]{27} \left(\cos\left(\frac{3\pi}{2}\right) + i \sin\left(\frac{3\pi}{2}\right) \right)$$

$$a = 0 \quad r = 27$$

$$= 3 \left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2} \right)$$

$$b = -27 \quad \theta = \frac{3\pi}{2}$$

$$= 3(0 + i)$$

$$r \text{ cis } \theta = 27 \text{ cis}\left(\frac{3\pi}{2}\right)$$

$$= 3i$$

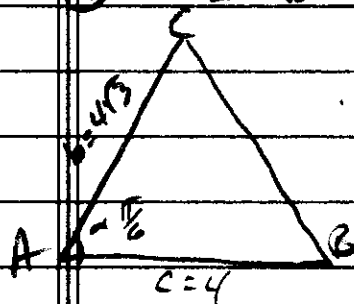
$$\begin{aligned} w_2 &= \sqrt[3]{27} \text{ cis}(\theta + 2\pi) \\ &= 3 \left(\cos\frac{2\pi + 3\pi}{2} + i \sin\frac{2\pi + 3\pi}{2} \right) \\ &= 3 \left(\cos\frac{5\pi}{2} + i \sin\frac{5\pi}{2} \right) \end{aligned}$$

$$\begin{aligned} w_3 &= \sqrt[3]{27} \left(\cos\frac{2\pi + 4\pi}{3} + i \sin\frac{2\pi + 4\pi}{3} \right) \\ &= 3 \left(\cos\frac{11\pi}{6} + i \sin\frac{11\pi}{6} \right) \\ &= 3 \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = \frac{3\sqrt{3}}{2} - \frac{3}{2}i \end{aligned}$$

$$\begin{aligned} \omega^2 &= 3 \left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2} \right) \\ &= 3(0 + i) \end{aligned}$$

3b

⑦ In ΔABC $\alpha = \frac{\pi}{6}$, $b = 4\sqrt{3}$, $c = 4$ Find possible values of β , b , γ



$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$= (4\sqrt{3})^2 + 4^2 - 2(4\sqrt{3})(4) \cos \frac{\pi}{6}$$

$$= 48 + 16 - 32\sqrt{3} \left(\frac{\sqrt{3}}{2} \right)$$

$$= 64 - 16(3)$$

$$a^2 = 16$$

$$a = 4$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$$

$$\sin \frac{\pi}{6} = \frac{\sin \beta}{4\sqrt{3}}$$

$$\sin \beta = \frac{4\sqrt{3} \cdot \sin \frac{\pi}{6}}{4} = \frac{\sqrt{3}}{2}$$

$$\beta_1 = 60^\circ \quad \beta_2 = 180^\circ - 60^\circ = 120^\circ$$

$$\gamma_1 = 180^\circ - \alpha - \beta_1 = 90^\circ$$

$$\gamma_2 = 180^\circ - \alpha - \beta_2 = 30^\circ$$

