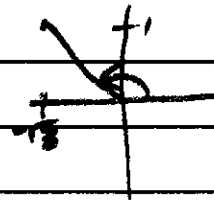
Trig Exam 3 Review

Conversion between $(x,y) \Leftrightarrow (r,\theta)$

1.a. $(-\sqrt{3}, 1) \to (r,\theta)$



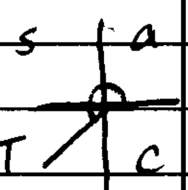
$\begin{cases} a = -\sqrt{3} \\ b = 1 \end{cases}$ $\quad r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$

$\tan\theta_r = \left|\frac{1}{-\sqrt{3}}\right| = \frac{\sqrt{3}}{3} \quad \theta_r = \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}$

$(r,\theta) = \left(2, \frac{5\pi}{6}\right)$

1.b. $\left(7, \frac{11\pi}{6}\right) \to (x,y)$



$\begin{cases} r = 7 \\ \theta = \frac{11\pi}{6} \end{cases} \Rightarrow$ $x = r\cos\frac{11\pi}{6} = 7(\pm\cos\theta_r) = 7\left(\cos\frac{\pi}{6}\right) = 7\cdot\frac{\sqrt{3}}{2} = \frac{7\sqrt{3}}{2}$

$y = r\sin\frac{11\pi}{6} = 7(\pm\sin\theta_r) = 7\left(-\sin\frac{\pi}{6}\right) = -7\cdot\frac{1}{2} = -\frac{7}{2}$

2. Compute $|7-4i| = \sqrt{7^2+4^2} = \sqrt{65}$

$|-\sqrt{31}\,i| = \sqrt{0^2 + (\sqrt{31})^2} = \sqrt{31}$

$|-\sqrt{31}| = \sqrt{(-\sqrt{31})^2} = \sqrt{31}$

3. Express $10\,\text{cis}\left(\frac{5\pi}{6}\right)$ as $a+bi$ form

$\begin{cases} r = 10 \\ \theta = \frac{5\pi}{6} \end{cases}$ $\quad \begin{cases} x = r\cos\frac{5\pi}{6} = 10\left(\pm\cos\frac{\pi}{6}\right) = -5\sqrt{3} \\ y = r\sin\frac{5\pi}{6}\,i = 10\left(\pm\sin\frac{\pi}{6}\right)i = 5i \end{cases}$ $\quad -5\sqrt{3}+5i$

4. Use DeMoivre's Thrm to change $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^6$ to $a+bi$ form

$\begin{cases} a = -\frac{1}{2} \\ b = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow$ $r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

$\tan\theta_r = \left|\frac{-\sqrt{3}/2}{-1/2}\right| = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3} \quad \theta_r = \frac{\pi}{3} \quad \theta = \frac{4\pi}{3}$

$= \left(1\,\text{cis}\left(\frac{4\pi}{3}\right)\right)^6 \qquad (z^n = r^n\,\text{cis}(n\cdot\theta))$

$= 1^6\,\text{cis}\left(6\cdot\frac{4\pi}{3}\right)$

$= \text{cis}(8\pi)$

$= \cos 8\pi + i\sin 8\pi$

$= 1 + i\cdot 0 = 1$

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⑤ $z_1 = 5i \qquad z_2 = \sqrt{2} + \sqrt{2}i$

Find $z_1 \cdot z_2$ $\qquad$ Find $\frac{z_1}{z_2}$

$z_1 \cdot z_2 = 5i(\sqrt{2} + \sqrt{2}i)$

$= 5\sqrt{2}i + 5\sqrt{2}i^2$

$= -5\sqrt{2} + 5\sqrt{2}i$

$\frac{5i}{\sqrt{2}+\sqrt{2}i} \cdot \frac{(\sqrt{2}-\sqrt{2}i)}{(\sqrt{2}-\sqrt{2}i)} = \frac{5\sqrt{2}i + 5\sqrt{2}}{2+2}$

$= \frac{5\sqrt{2}}{4} + \frac{5\sqrt{2}}{4}i$

2 problems of this type

⑥ Find the cubic roots of $-27i$, Put answer in $a+bi$ form

$z = -27i$

$a = 0 \qquad r = 27$

$b = -27 \qquad \theta = \frac{3\pi}{2}$

$r\,cis\,\theta = 27\,cis(\frac{3\pi}{2})$

$w_1 = \sqrt[3]{27}(\cos(\frac{\frac{3\pi}{2}}{3}) + i\sin(\frac{\frac{3\pi}{2}}{3}))$

$= 3(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

$= 3(0 + i)$

$= \boxed{3i}$

$w_2 = \sqrt[3]{27}\,cis(\frac{\theta + 2\pi}{3})$

$= 3(\cos\frac{\frac{3\pi}{2}+2\pi}{3} + i\sin\frac{\frac{3\pi}{2}+2\pi}{3})$

$= 3(\cos\frac{7\pi}{2/3} + i\sin\frac{7\pi}{2/3})$

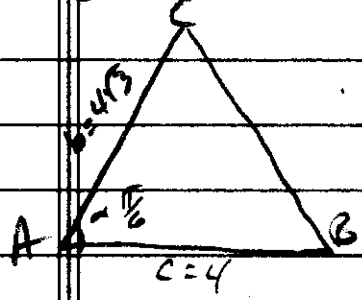
$w_2 =$
$3(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$
$3(0+i)$
$3i$

$w_3 = \sqrt[3]{27}(\cos\frac{\frac{3\pi}{2}+4\pi}{3} + i\sin\frac{\frac{3\pi}{2}+4\pi}{3})$

$= 3(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6})$

$= 3(\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \boxed{\frac{3\sqrt{3}}{2} - \frac{3}{2}i}$

⑦ In $\triangle ABC$ $\alpha = \frac{\pi}{6}$, $b = 4\sqrt{3}$, $c = 4$ Find possible values of β, b, γ



$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$= (4\sqrt{3})^2 + 4^2 - 2(4\sqrt{3})(4)\cos\frac{\pi}{6}$

$= 48 + 16 - 32\sqrt{3}(\frac{\sqrt{3}}{2})$

$= 64 - 16(3)$

$a^2 = 16$

$a = 4$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$

$\frac{\sin\frac{\pi}{6}}{4} = \frac{\sin\beta}{4\sqrt{3}}$

$\sin\beta = \frac{4\sqrt{3}\cdot\sin\frac{\pi}{6}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\beta_1 = 60° \qquad \beta_2 = 180° - 60° = 120°$

$\gamma_1 = 180° - \alpha - \beta_1 = 90°$

$\gamma_2 = 180° - \alpha - \beta_2 = 30°$

Inrg Exam #3 Review

8. $4+8+12+\dots+4n = 2n(n+1)$

① $n=1$ $LS=4$ $RS=2(1)(1+1)$

$RS=4$ $LS=RS$ ✓

② assume $n=k$ $4+8+12+\dots+4k=2k(k+1) = 2k(k+1)$

③ $n=k+1$ $LS=\underbrace{4+8+12+\dots+4k}+4(k+1)$ $RS=2(k+1)((k+1)+1)$

$2k(k+1)+4(k+1)$ $=2(k+1)(k+2)$

$(2k+4)(k+1)$ $=(k+1)(2k+2)$

$LS=RS$

④ $4+8+12+\dots+4n=2n(n+1)$ for all $n \geq 1$

9. $4+10+16+\dots+(6n-2)=n(3n+1)$

① $n=1$ $LS=6(1)-2=4$ $RS=1(3(1)+1)=4$ $LS=RS$ ✓

② assume $n=k$ $4+10+16+\dots+(6k-2)=k(3k+1)$

③ $n=k+1$ $4+10+16+\dots+(6k-2)+6(k+1)-2=k(3k+1)$

$LS=\underbrace{4+10+16+\dots+6k-2}+6(k+1)-2$ $RS=(k+1)(3(k+1)+1)$

$k(3k+1)+6(k+1)-2$ $=(k+1)(3k+4)$

$k(3k+1)+6k+4$

$3k^2+k+6k+4$ $LS=RS$ ✓

$3k^2+7k+4$

$(3k+4)(k+1)$

④ So $4+10+16+\dots+(6n-2)=n(3n+1)$ for all $n \geq 1$